



Partial Orders Fit For Work

Jörg Desel

FernUniversität in Hagen

based on a talk at the
Carl Adam Petri Memorial Symposium
last week in Berlin

Partial Orders Fit For Work

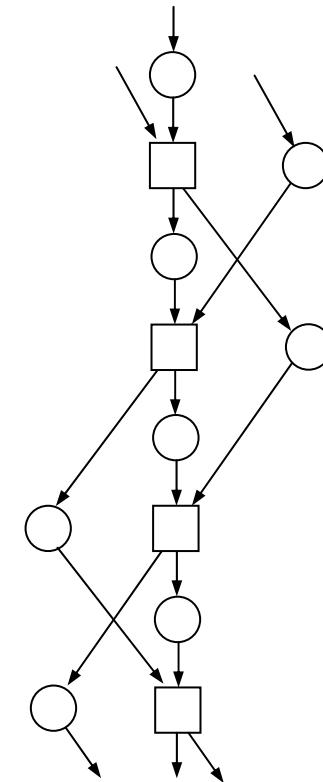
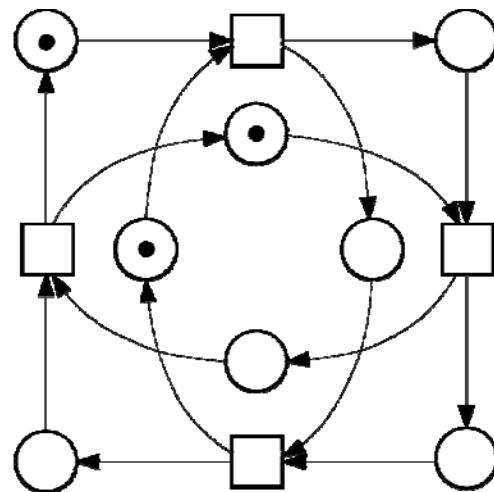
PART I:

**A proof using partial orders
(occurrence nets)
work done in 1988 at GMD**

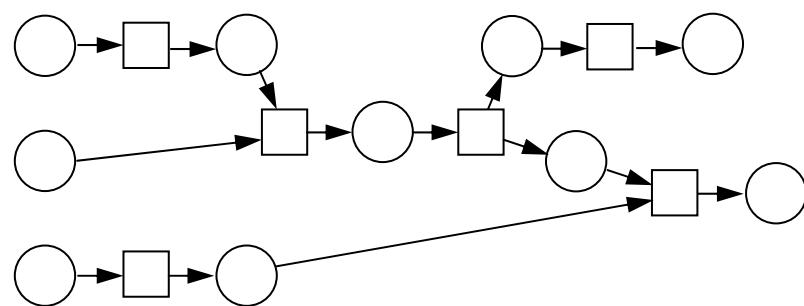
PART II:

**Process Model Verification, Validation and Synthesis
Based on Partial Orders
(VIPtool)
work done from 1996 until today**

PART I: A proof using partial orders (occurrence nets) work done in 1988 at GMD

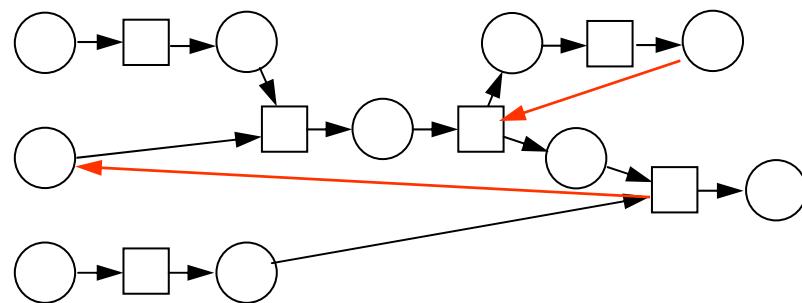


An occurrence net (B, E, K)



An occurrence net (B, E, K)

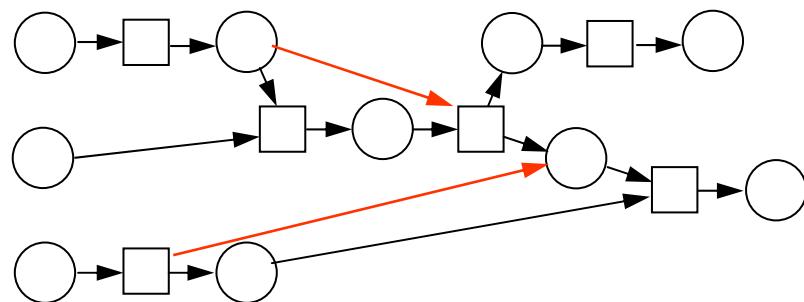
acyclic



An occurrence net (B, E, K)

acyclic

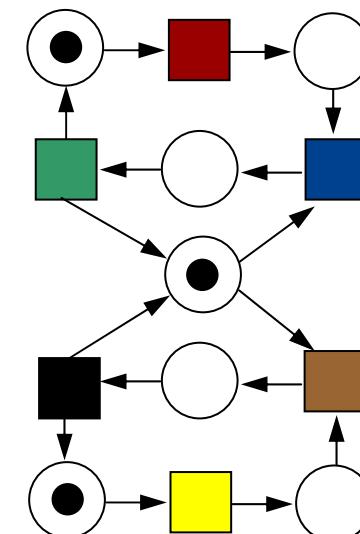
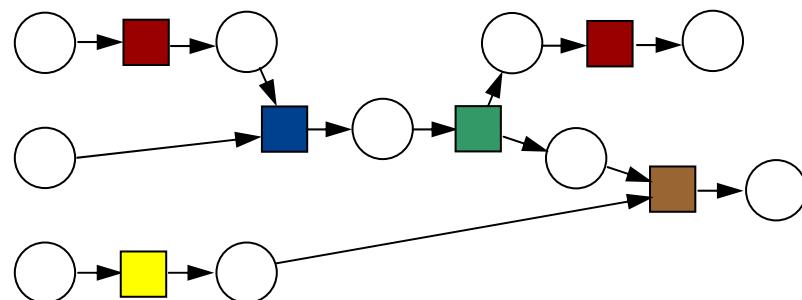
places unbranched



An occurrence net (B, E, K)

of a Petri net (S, T, F)

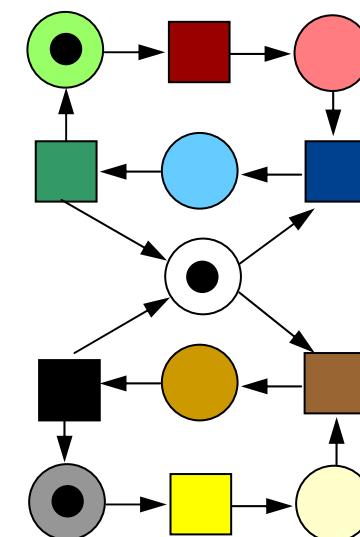
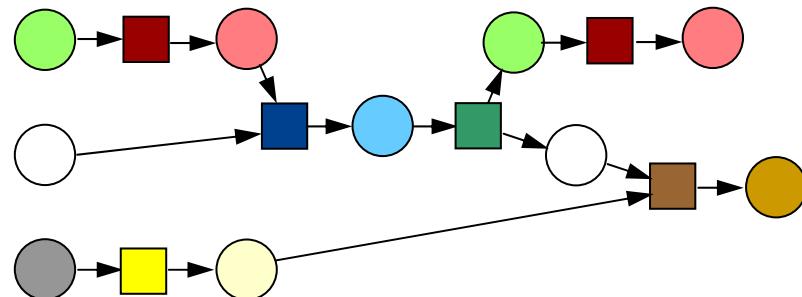
acyclic
places unbranched
maps to the Petri net



An occurrence net (B,E,K)

of a Petri net (S, T, F)

acyclic
places unbranched
maps to the Petri net



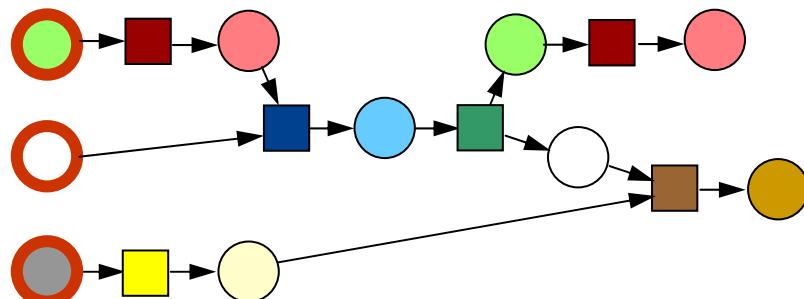
An **occurrence net** (B, E, K)

acyclic

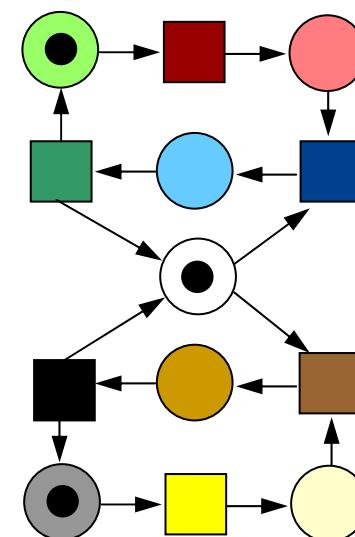
places unbranched

maps to the Petri net

minimal places map to tokens



of a **Petri net** (S, T, F)



An occurrence net (B,E,K)

of a Petri net (S, T, F)

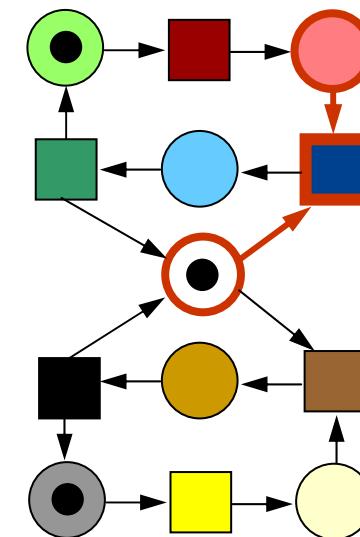
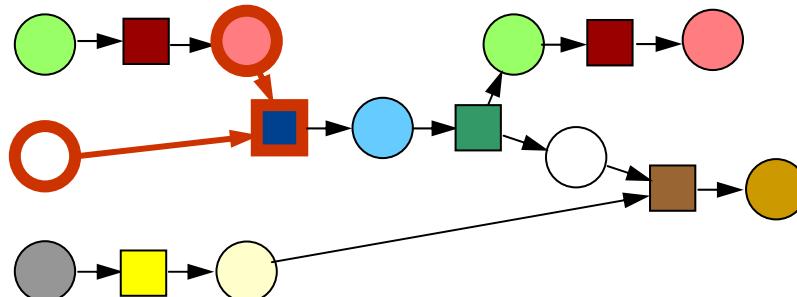
acyclic

places unbranched

maps to the Petri net

minimal places map to tokens

presets of transitions map to presets of transitions



An occurrence net (B,E,K)

acyclic

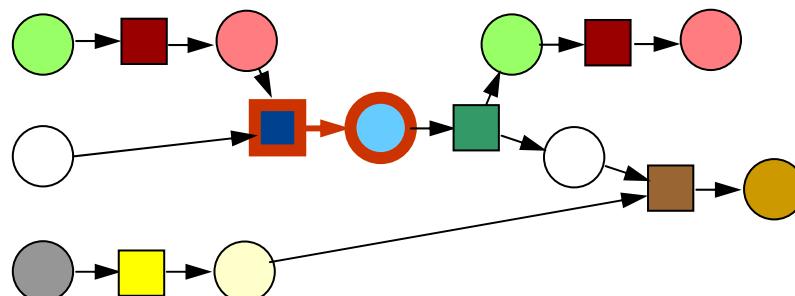
places unbranched

maps to the Petri net

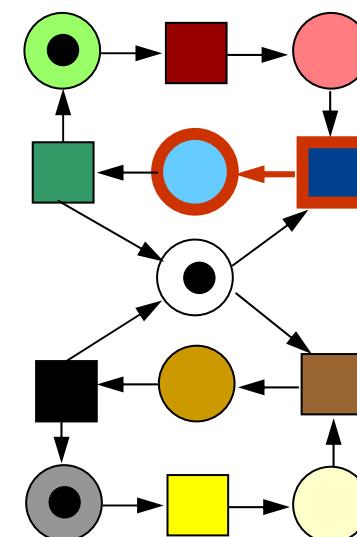
minimal places map to tokens

presets of transitions map to presets of transitions

postsets of transitions map to postsets of transitions

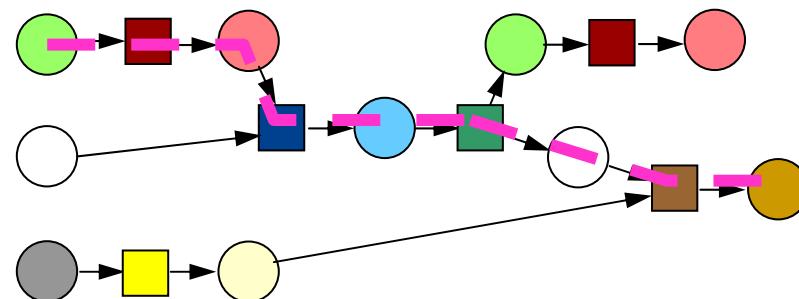


of a Petri net (S,T,F)



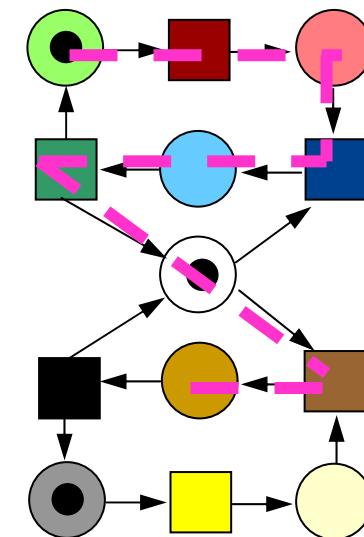
An occurrence net (B, E, K)

Lemma: paths map to



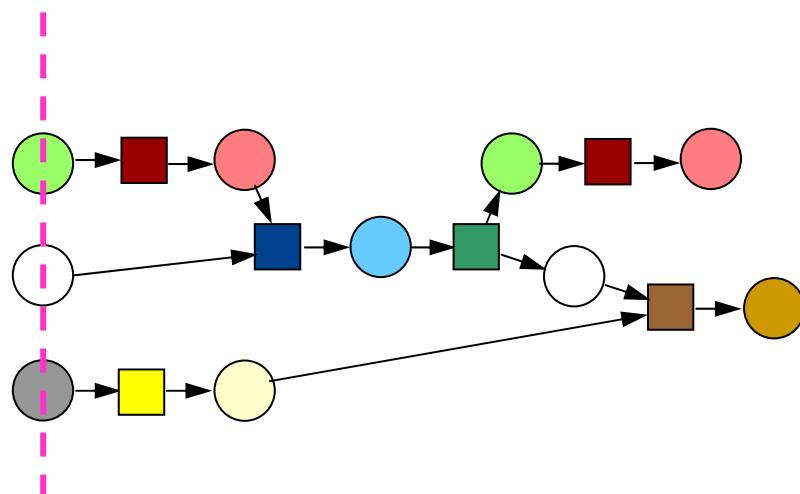
of a Petri net (S, T, F)

paths



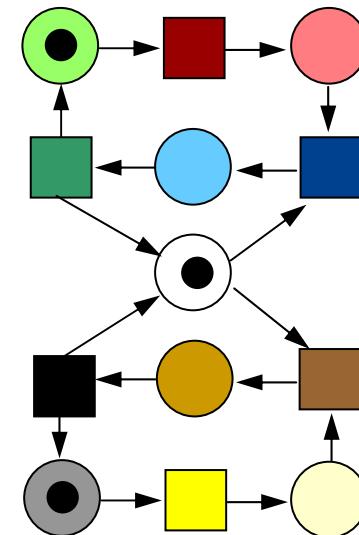
An occurrence net (B, E, K)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to



of a Petri net (S, T, F)

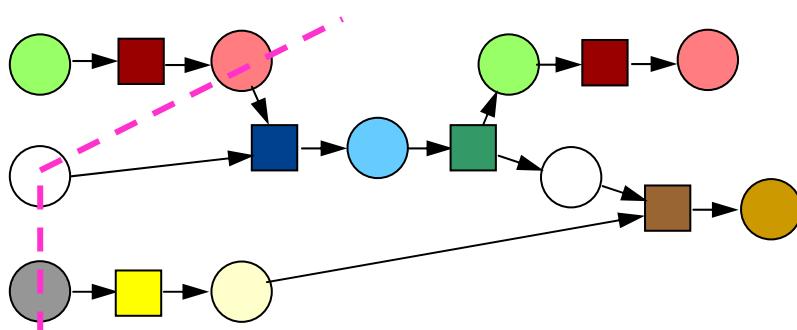
reachable markings



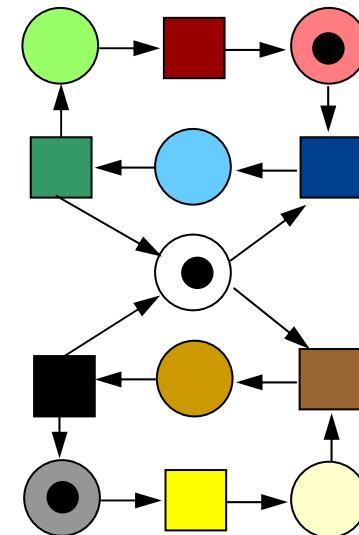
An occurrence net (B,E,K)

of a Petri net (S, T, F)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to

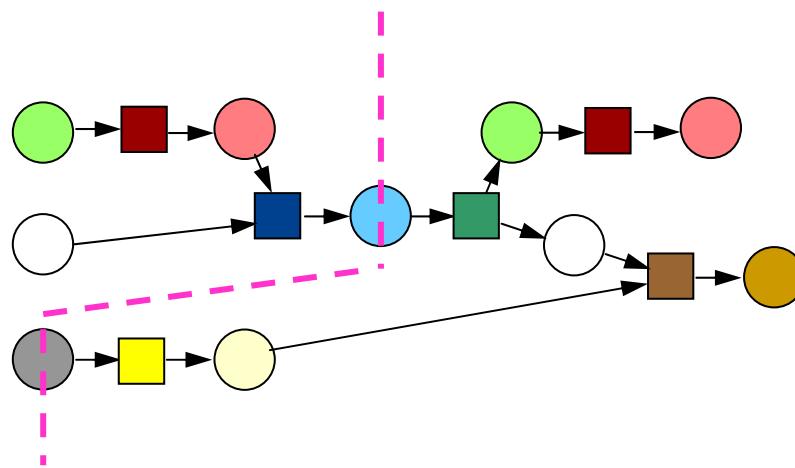


reachable markings



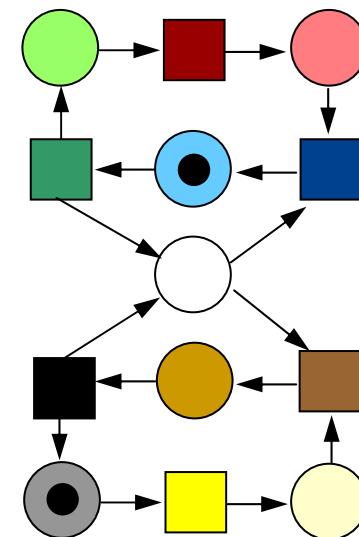
An occurrence net (B, E, K)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to



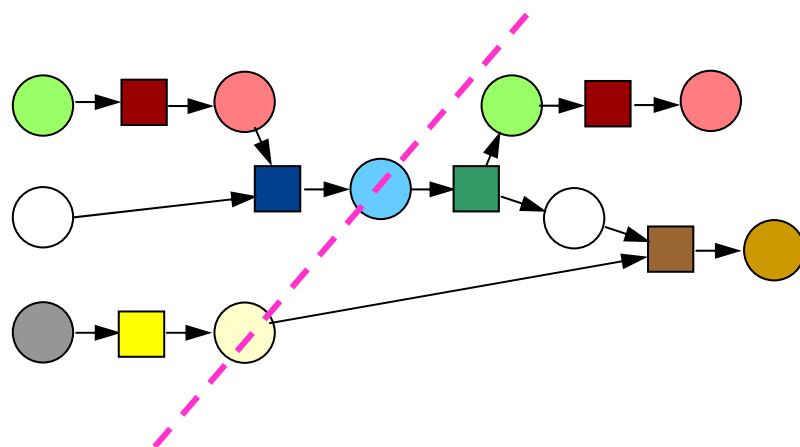
of a Petri net (S, T, F)

reachable markings



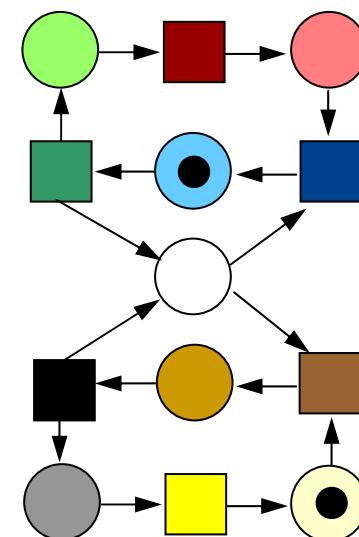
An occurrence net (B, E, K)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to



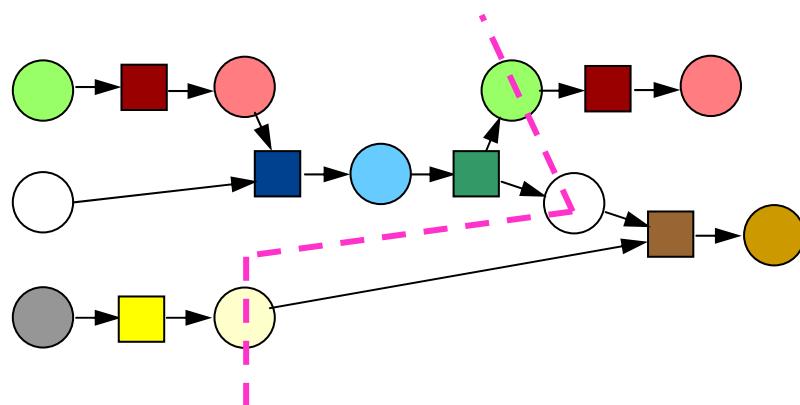
of a Petri net (S, T, F)

reachable markings



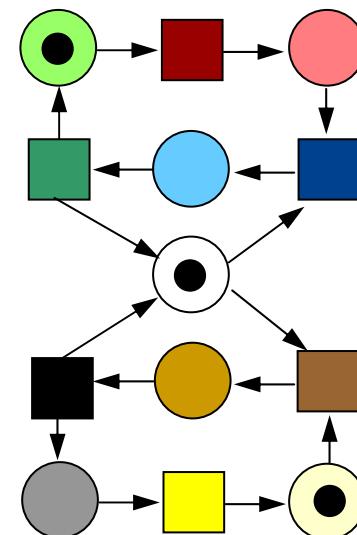
An occurrence net (B, E, K)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to



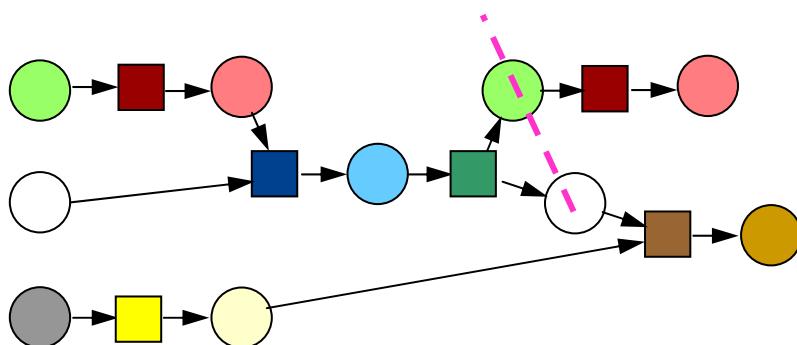
of a Petri net (S, T, F)

reachable markings



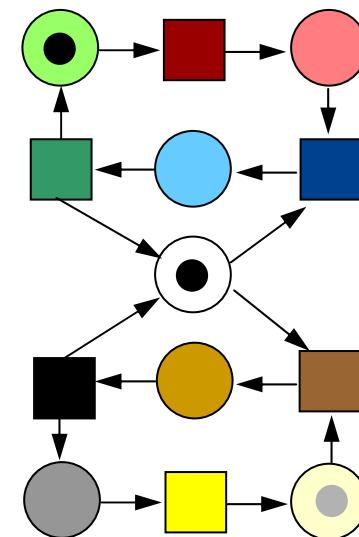
An occurrence net (B, E, K)

**Corollary: finite co-sets
(sets of mutually concurrent places)
map to**



of a Petri net (S, T, F)

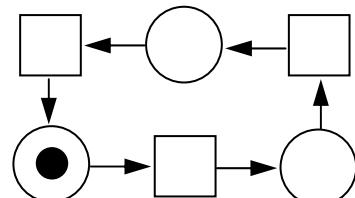
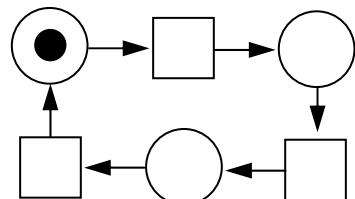
reachable sub-markings



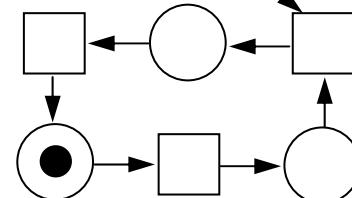
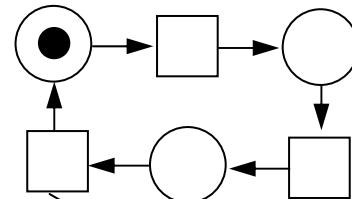
Theorem: each connected live and bounded Petri net is strongly connected

Theorem: each **connected** live and bounded Petri net is **strongly connected**

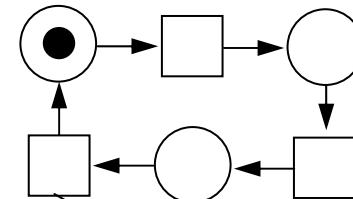
not connected



connected



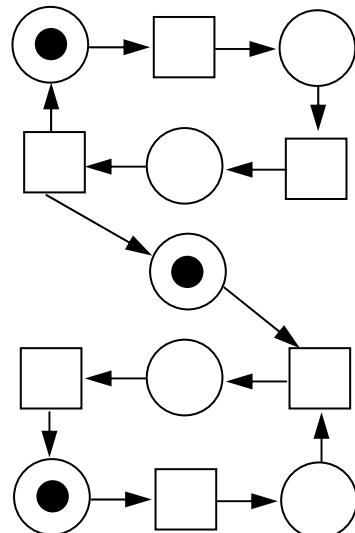
strongly connected



Theorem: each connected **live and **bounded** Petri net is strongly connected**

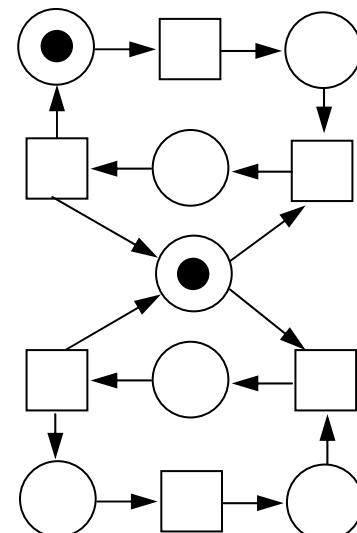
live, not bounded

each transition can
always occur again

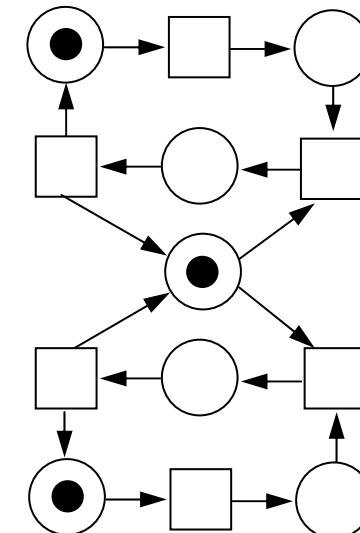


bounded, not live

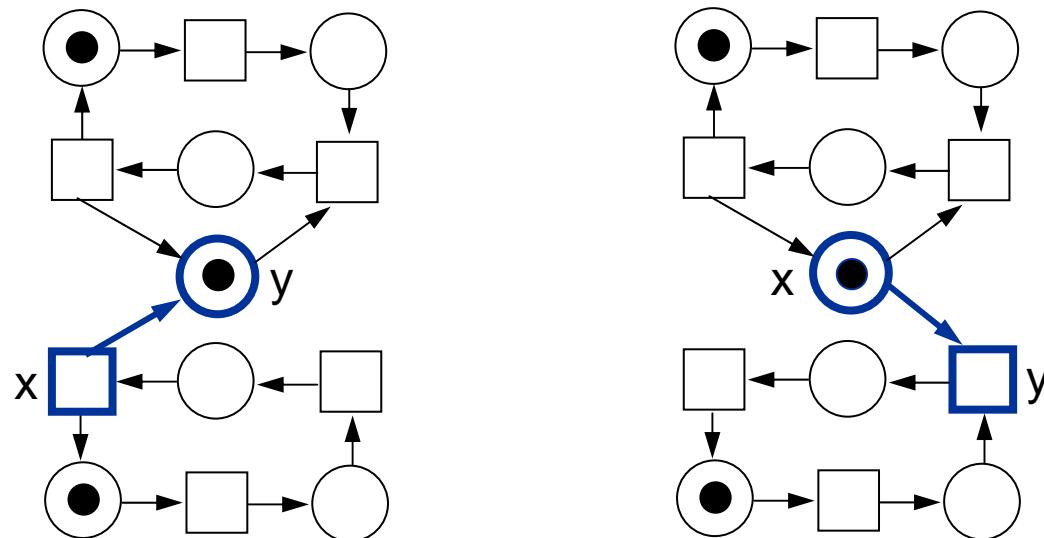
each place has a bound
(maximal number of tokens)



live and bounded



Theorem: each connected live and bounded Petri net is strongly connected



Lemma: if a net is connected but not strongly connected then for some arc (x,y) there is no directed path from y to x

Corollary: if, in a connected net, for each arc (x,y) there is a path from y to x , then the net is strongly connected

Theorem: each connected live and bounded Petri net is strongly connected

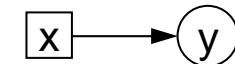
Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

We will show that there is a path from y to x .

Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

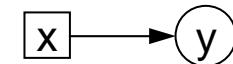


Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).



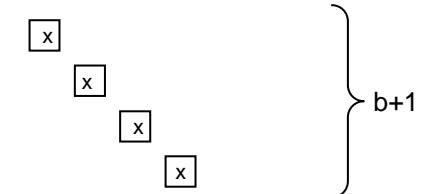
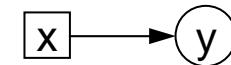
Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .
(exists, because the net is live).



Theorem: each connected live and bounded Petri net is strongly connected

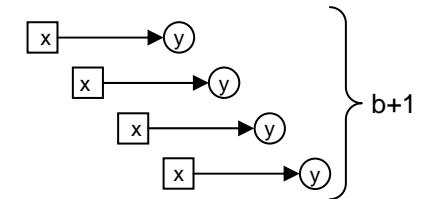
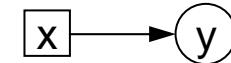
Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .
(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

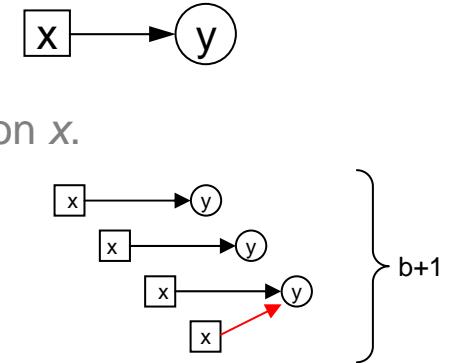
Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .

(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.

Since places in occurrence nets are not branched,
all these occurrences of y are distinct.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .

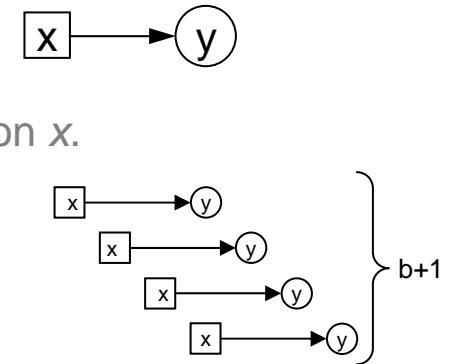
(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.

Since places in occurrence nets are not branched,
all these occurrences of y are distinct.

Since b is its bound, the place y never carries more than b tokens.

Hence no co-set contains all $b+1$ occurrences of y .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

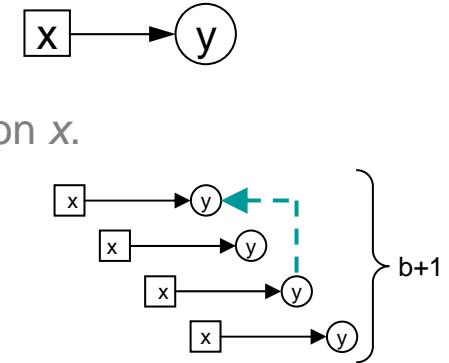
Assume an occurrence net with $b+1$ occurrences of the transition x .
(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.

Since places in occurrence nets are not branched,
all these occurrences of y are distinct.

Since b is its bound, the place y never carries more than b tokens.
Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .
(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.

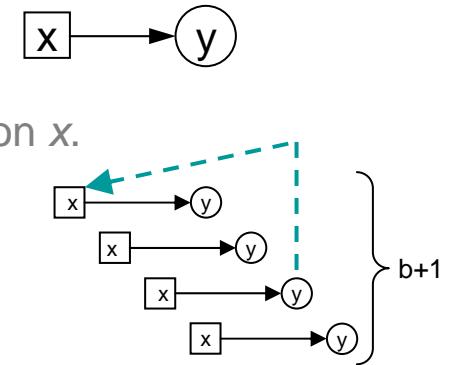
Since places in occurrence nets are not branched,
all these occurrences of y are distinct.

Since b is its bound, the place y never carries more than b tokens.
Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.

Again since places in occurrence nets are not branched,
this path goes through an occurrence of x .

So there is a path from an occurrence of y to an occurrence of x .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b+1$ occurrences of the transition x .
(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.

Since places in occurrence nets are not branched,
all these occurrences of y are distinct.

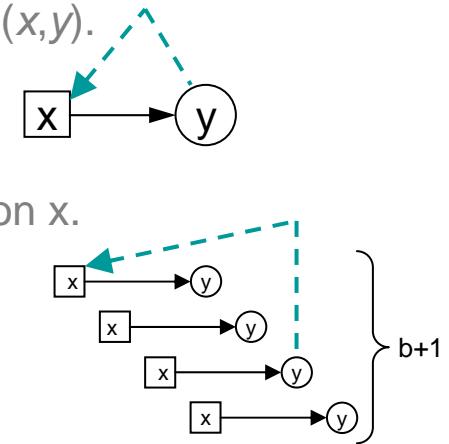
Since b is its bound, the place y never carries more than b tokens.
Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.

Again since places in occurrence nets are not branched,
this path goes through an occurrence of x .

So there is a path from an occurrence of y to an occurrence of x .

Since paths are mapped to paths, there is a path from y to x .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 2: x is a place and y is a transition.

Let b be the bound of x (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition y .
(exists, because the net is live).

Since presets of the occurrences of y are respected,
each occurrence of y has an occurrence of x in its preset.

Since places in occurrence nets are not branched,
all these occurrences of x are distinct.

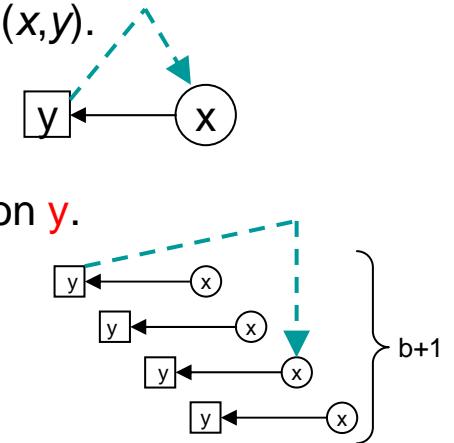
Since b is its bound, the place x never carries more than b tokens.
Hence no co-set contains all $b+1$ occurrences of x .

So at least two of these occurrences are connected by a path.

Again since places in occurrence nets are not branched,
this path goes through an occurrence of y .

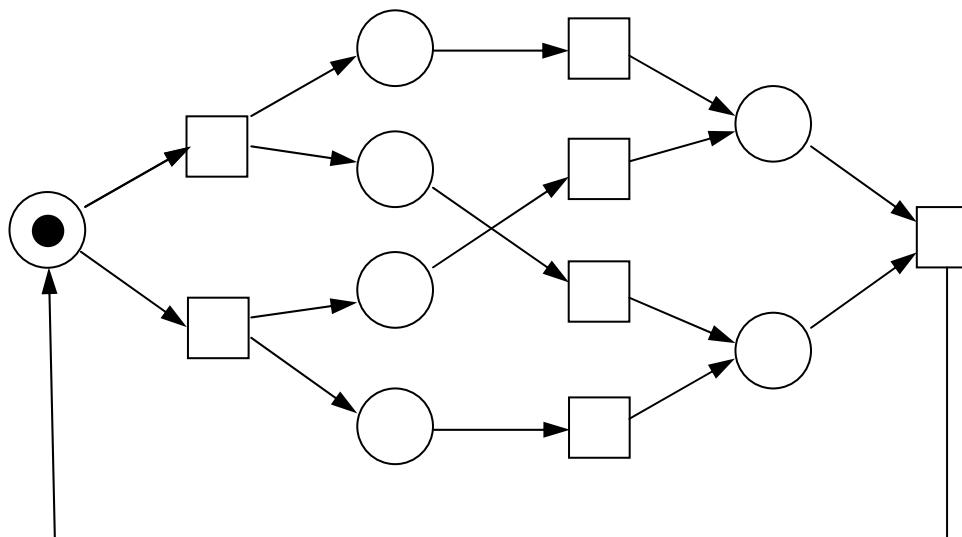
So there is a path from an occurrence of y to an occurrence of x .

Since paths are mapped to paths, there is a path from y to x .



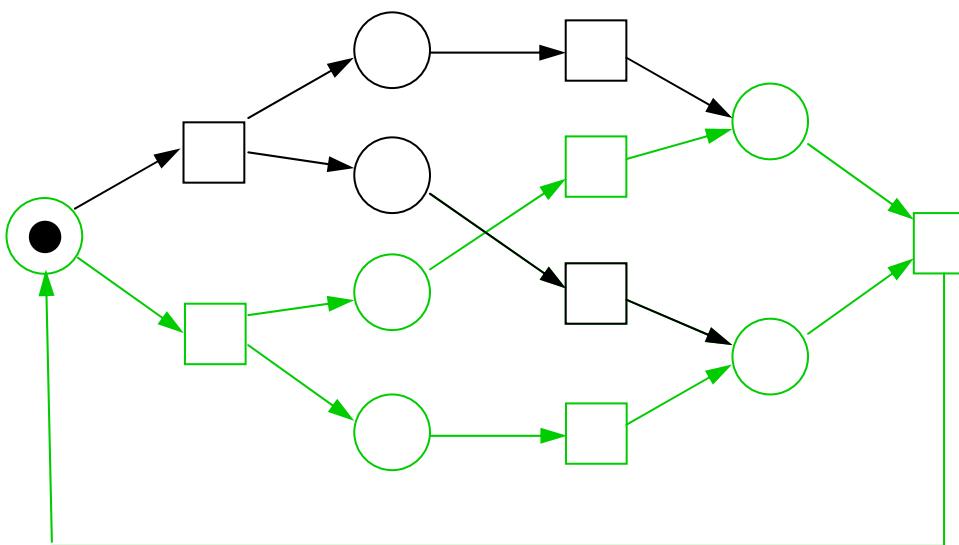
Further Theorems, using the same idea:

each live and bounded extended free-choice net is covered by T-components



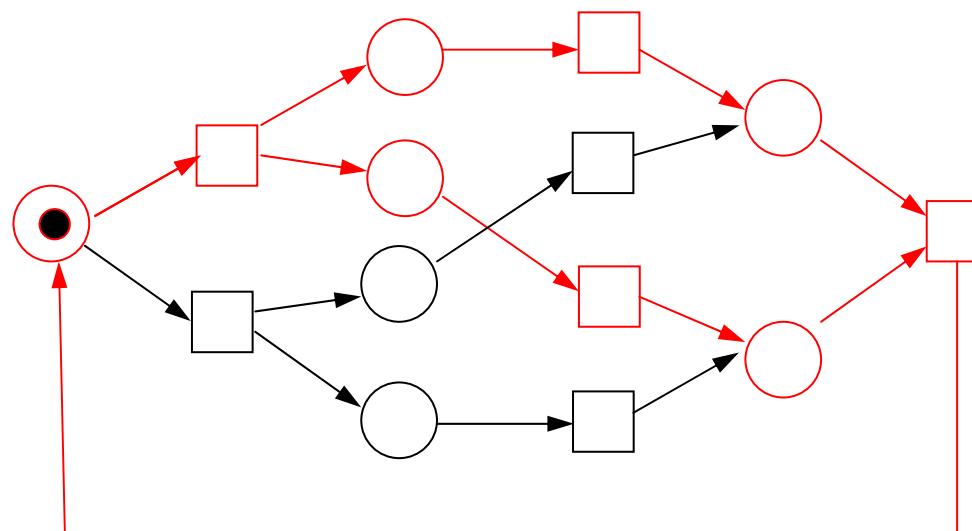
Further Theorems, using the same idea:

each live and bounded extended free-choice net is covered by T-components



Further Theorems, using the same idea:

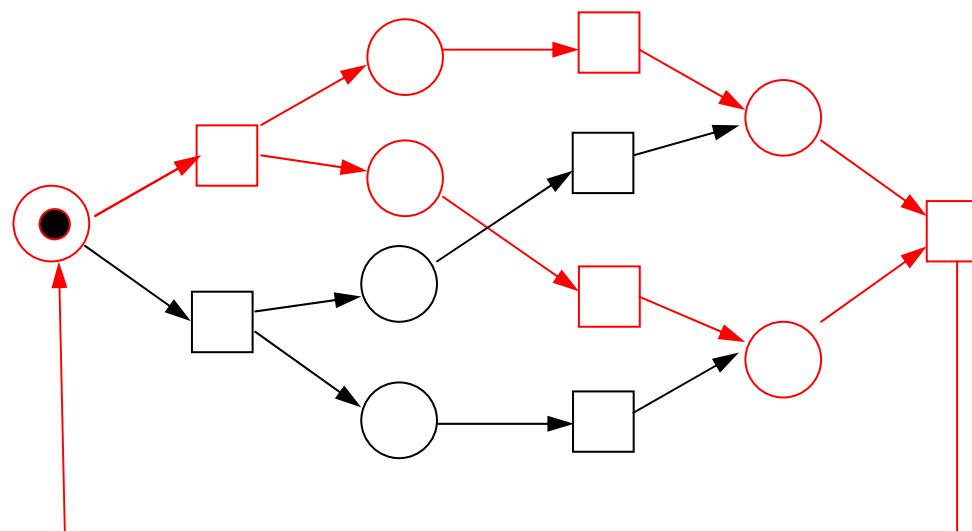
each live and bounded **extended free-choice net** is covered by T-components



Further Theorems, using the same idea:

each live and bounded **extended free-choice net** is covered by T-components

each **T-invariant** is realizable, i.e., corresponds to a cyclic process



PART II:

Process Model Synthesis From Partial Orders (VIPtool)

PART II:

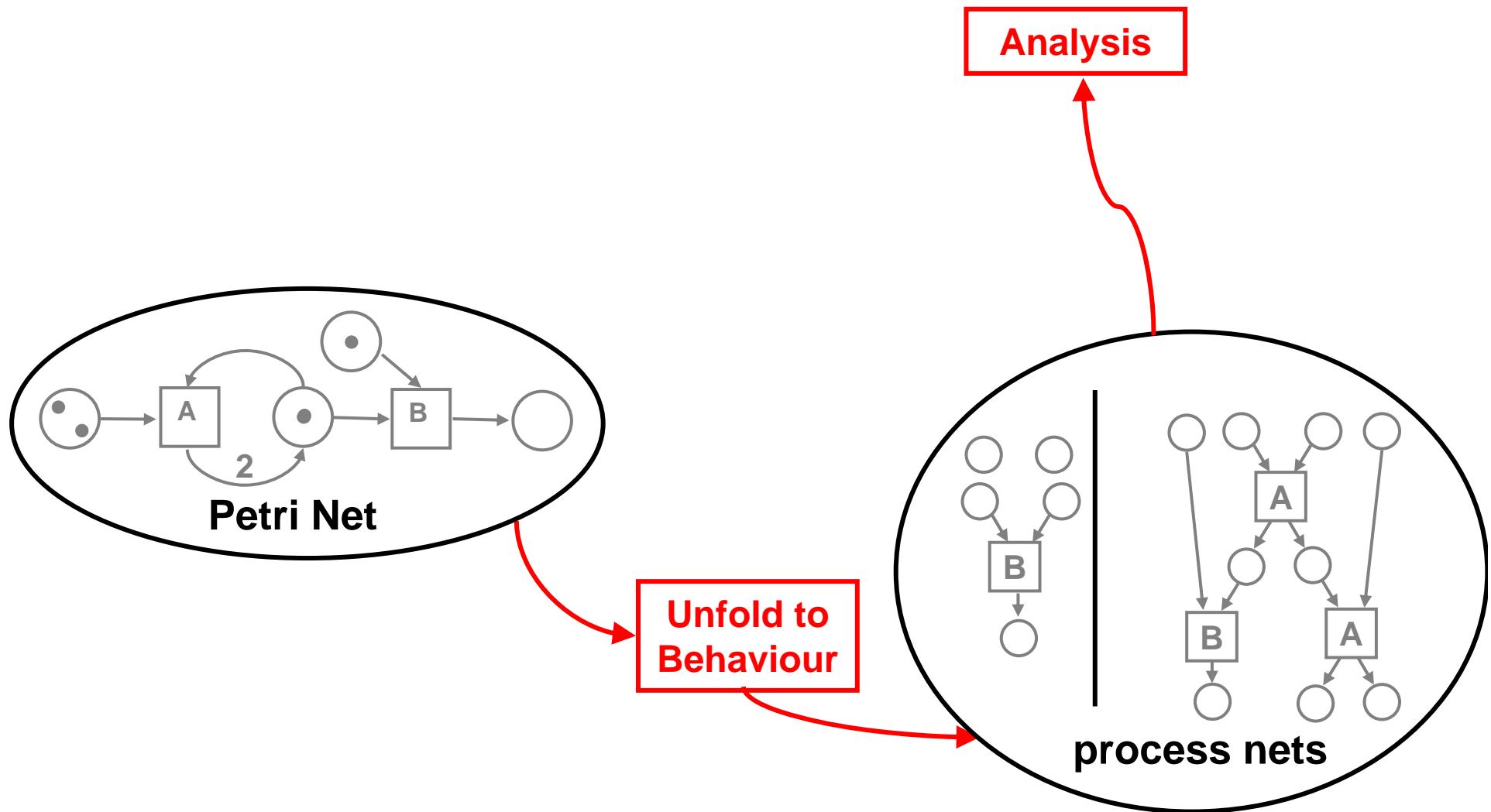
Process Model Synthesis From Partial Orders (VIPtool)

**VIPtool was originally created in 1996 -1998
by my group and the group of Andreas Oberweis,
Institute AIFB, University of Karlsruhe
(Carl-Adam-Petri award !!)**

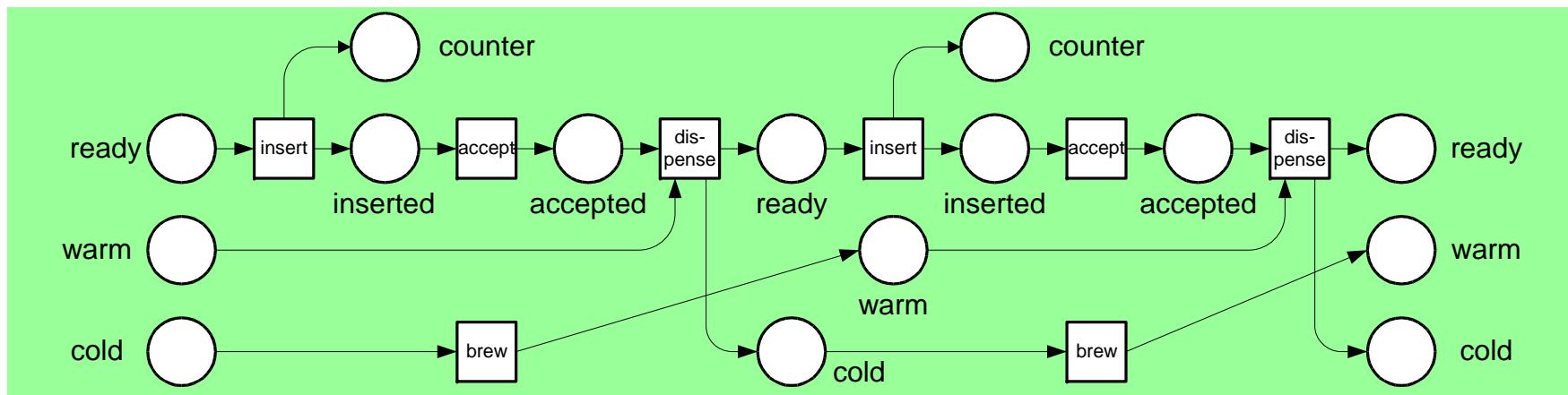
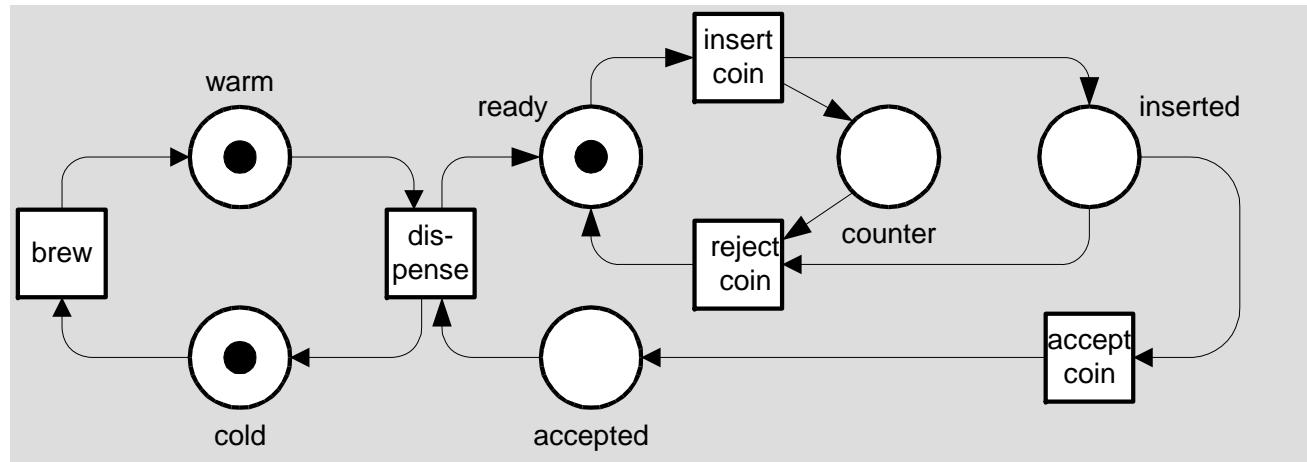
Simulation for Analysis

means generation of runs.

- **sequential runs** (occurrence sequences),
combined with graphical animation
 ⇒ the usual approach
- **non-sequential, causal runs** (process nets)
 ⇒ the VIP-approach



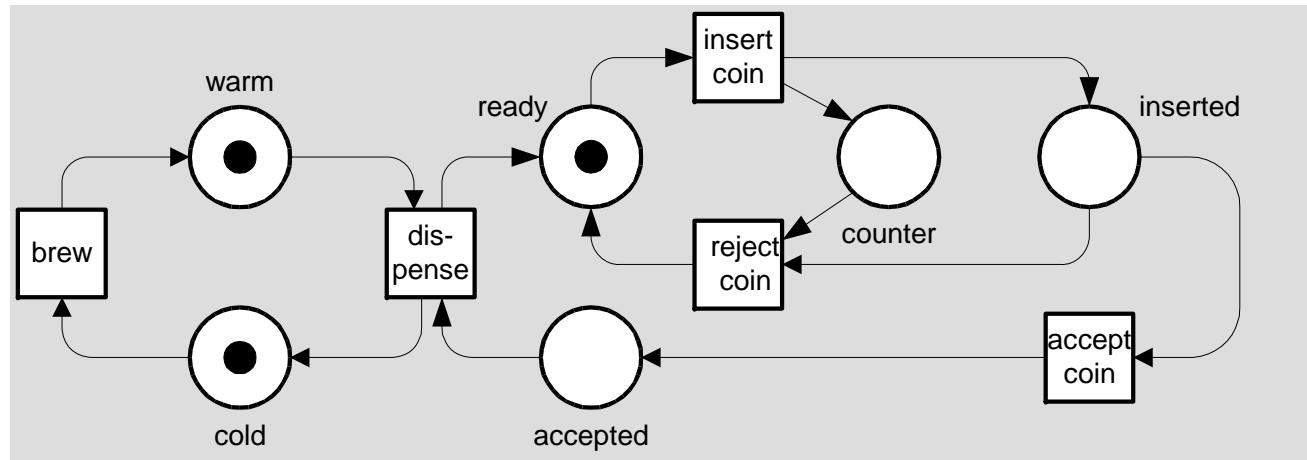
A Coffee Machine with one Process



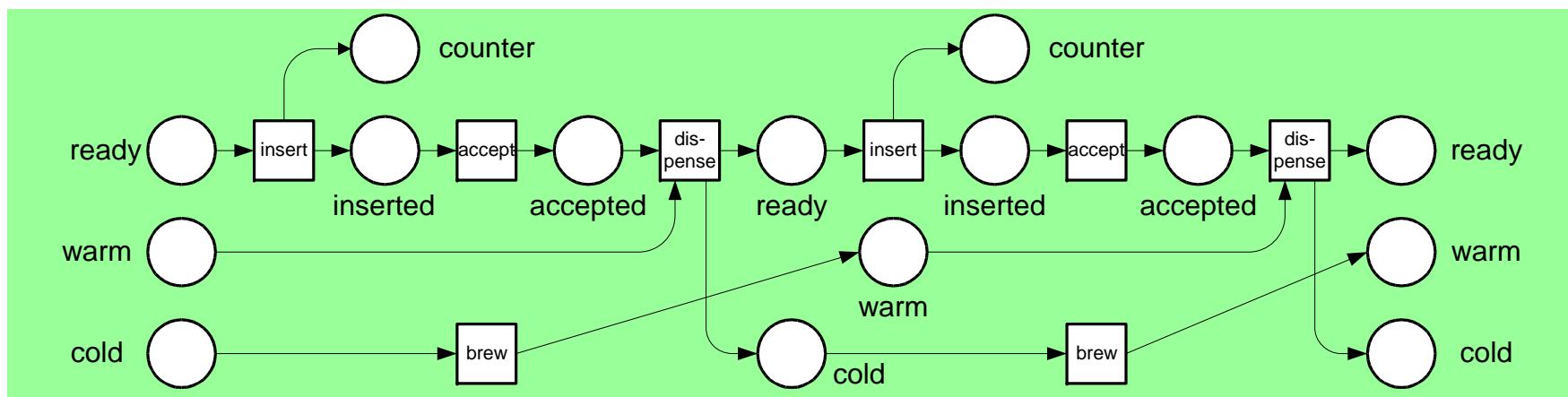
Efficiency

- every occurrence sequence is represented by an occurrence sequence of a process net
- every occurrence sequence of a process net corresponds to an occurrence sequence of the model
- the number of process nets exceeds the number of occurrence sequences significantly
(exponential in the degree of parallelism)

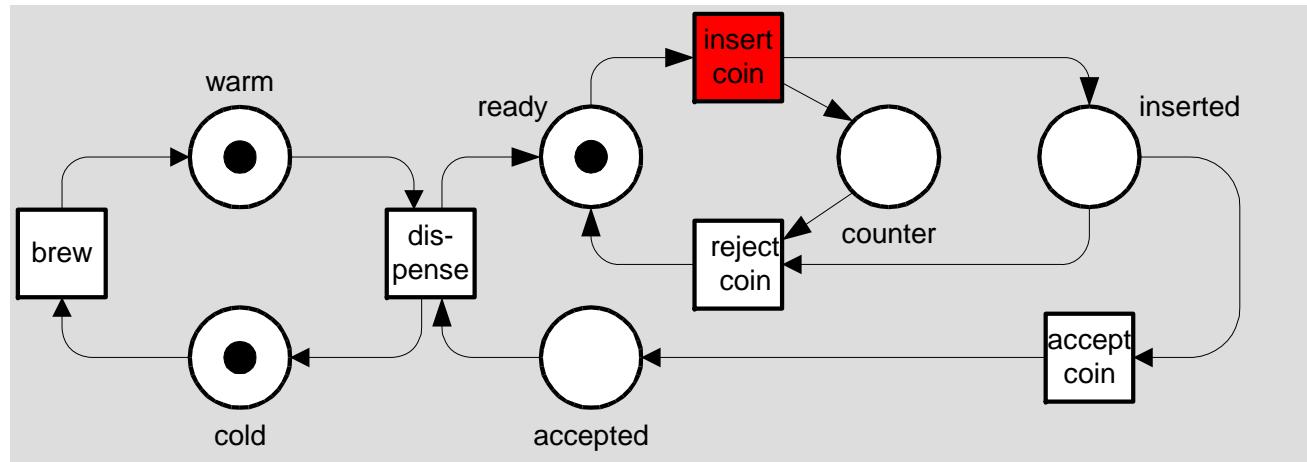
Advantage 1: Efficiency



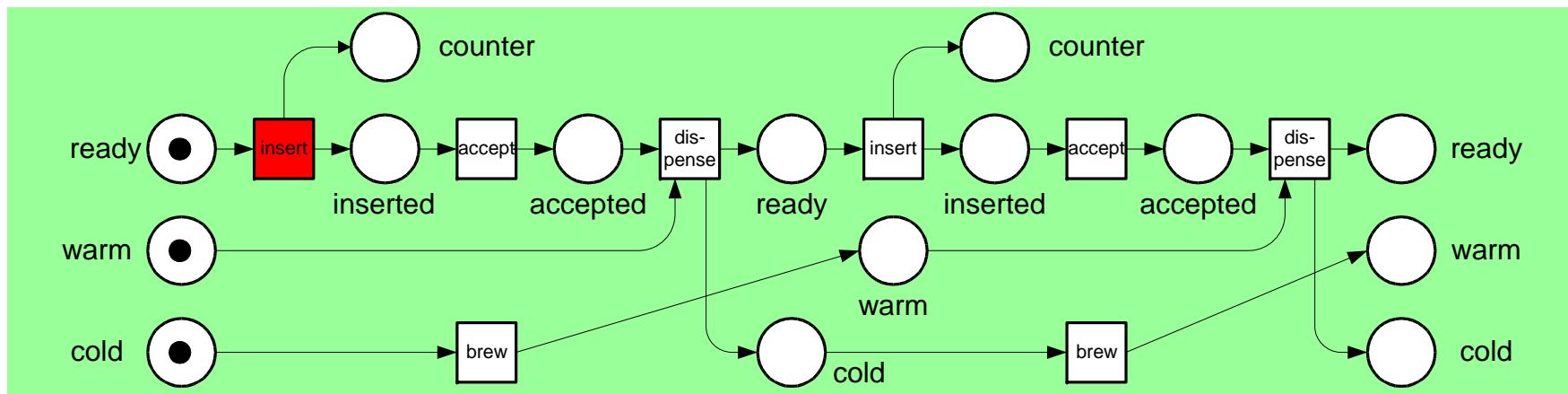
The process net represents 21 different occurrence sequences



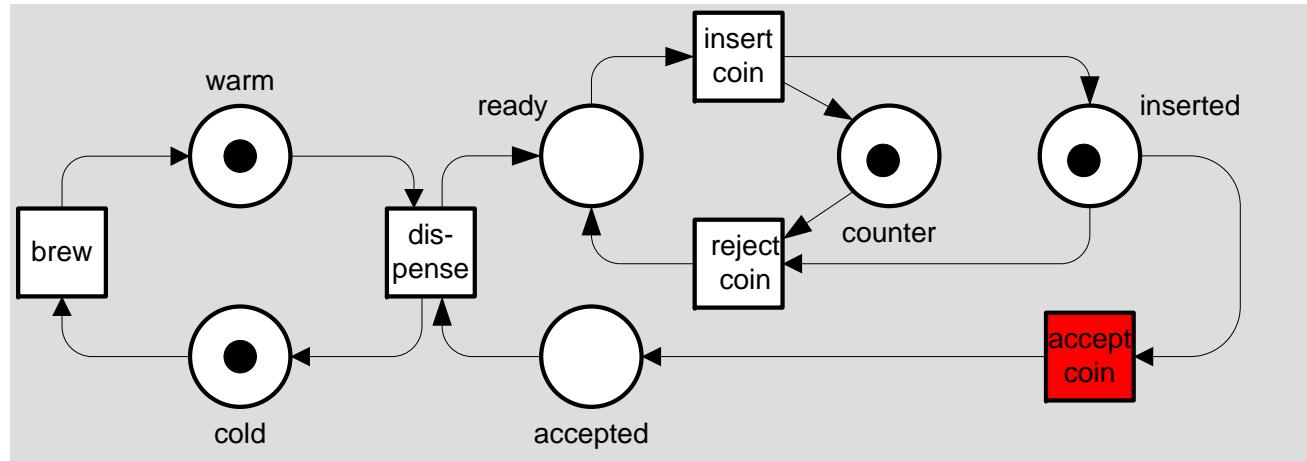
One common occurrence sequence



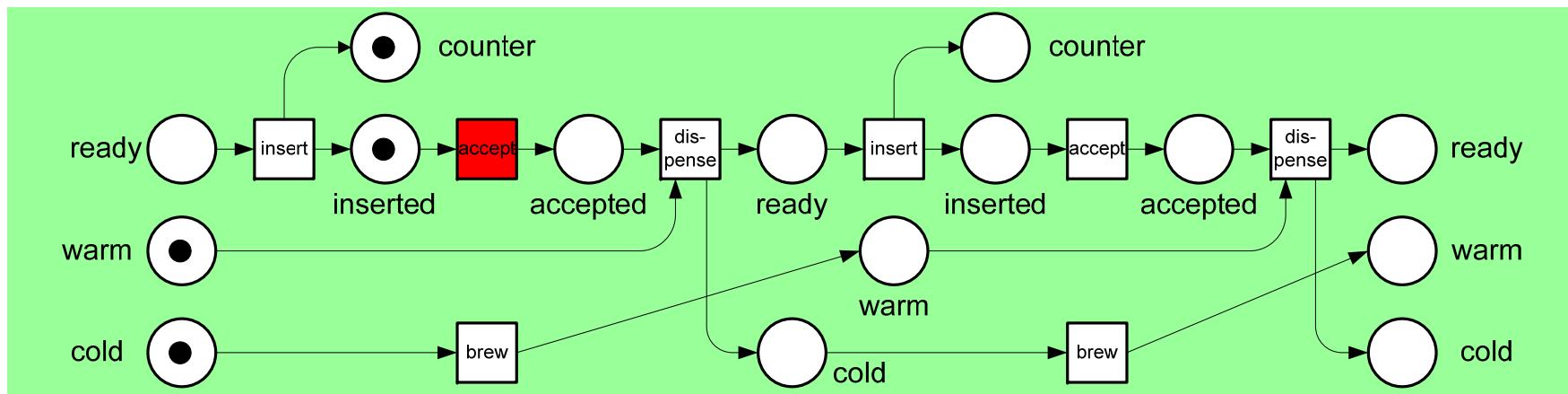
insert



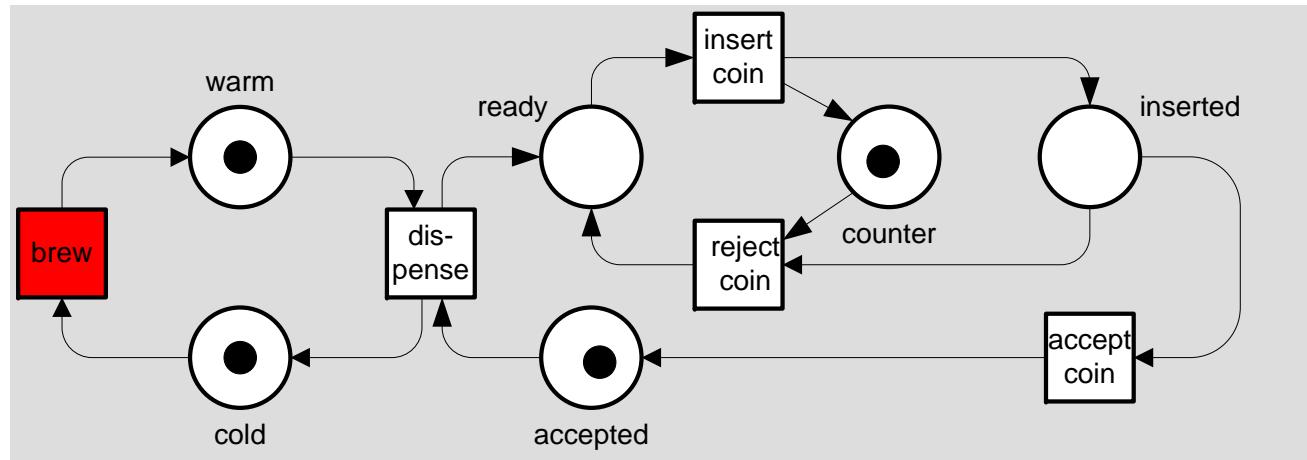
One common occurrence sequence



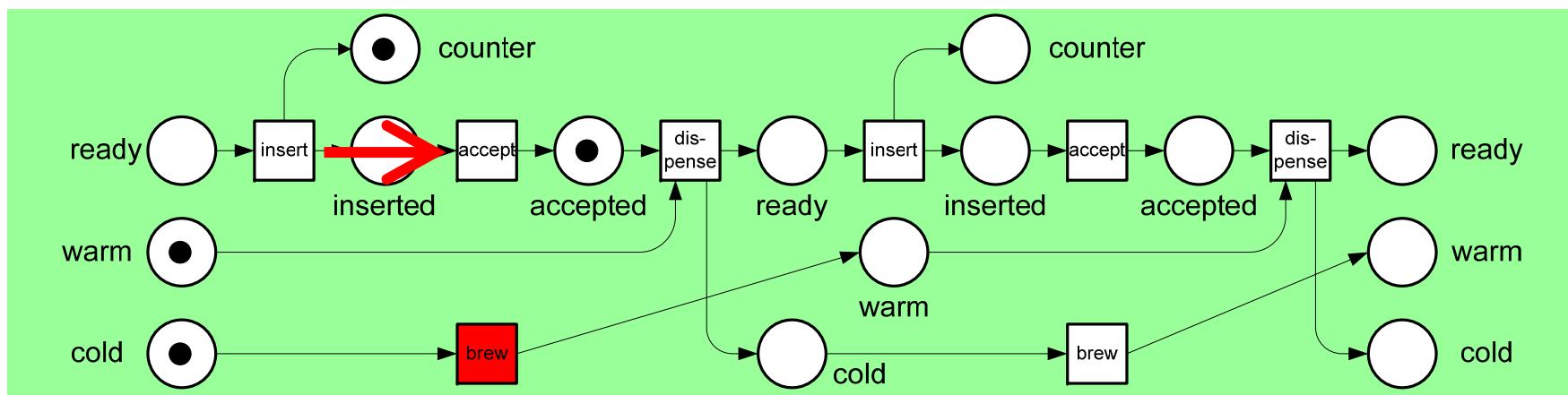
insert, accept



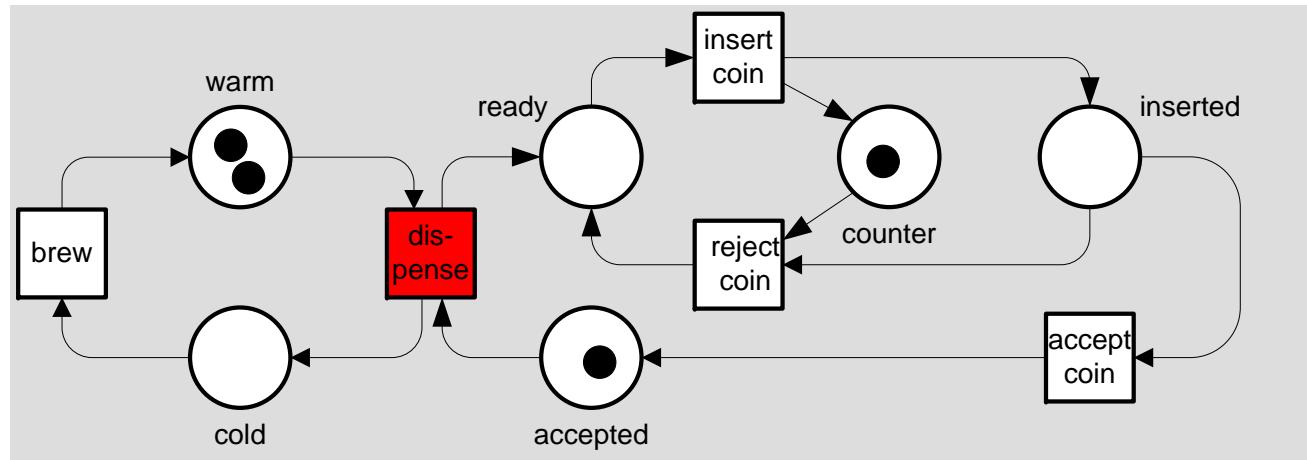
One common occurrence sequence



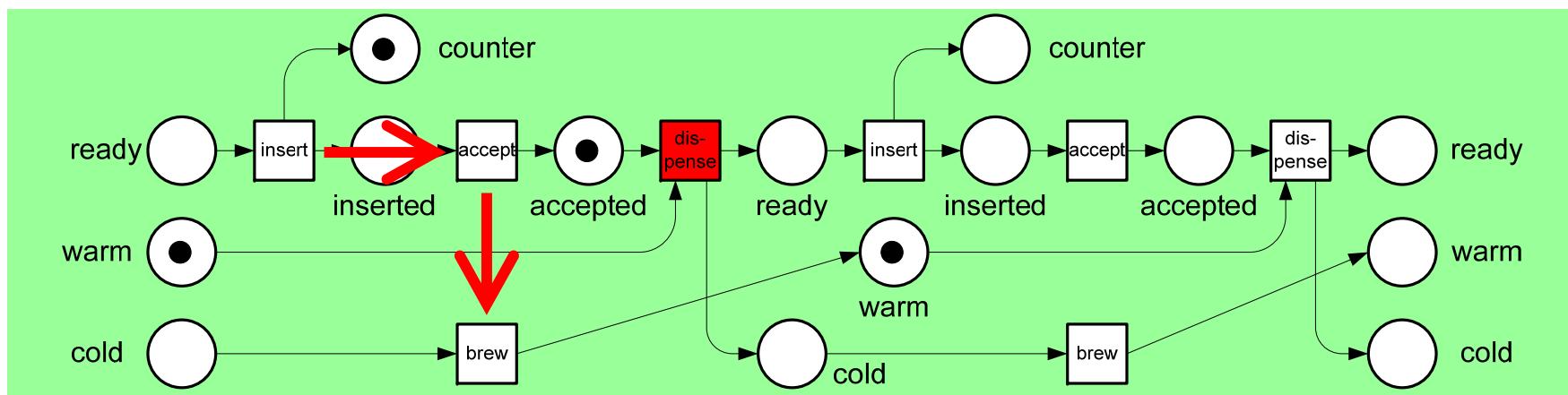
insert, accept, brew



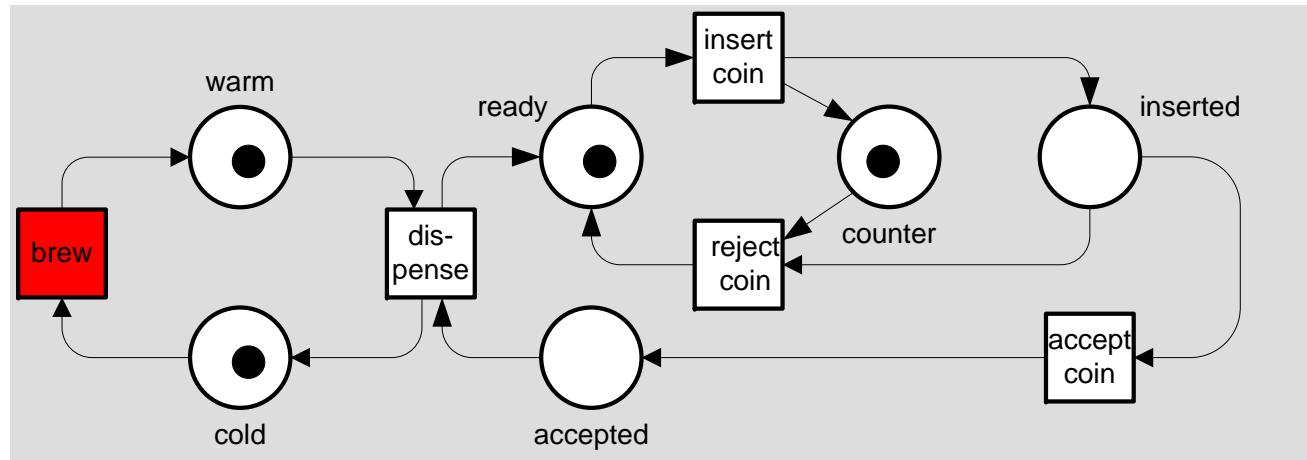
One common occurrence sequence



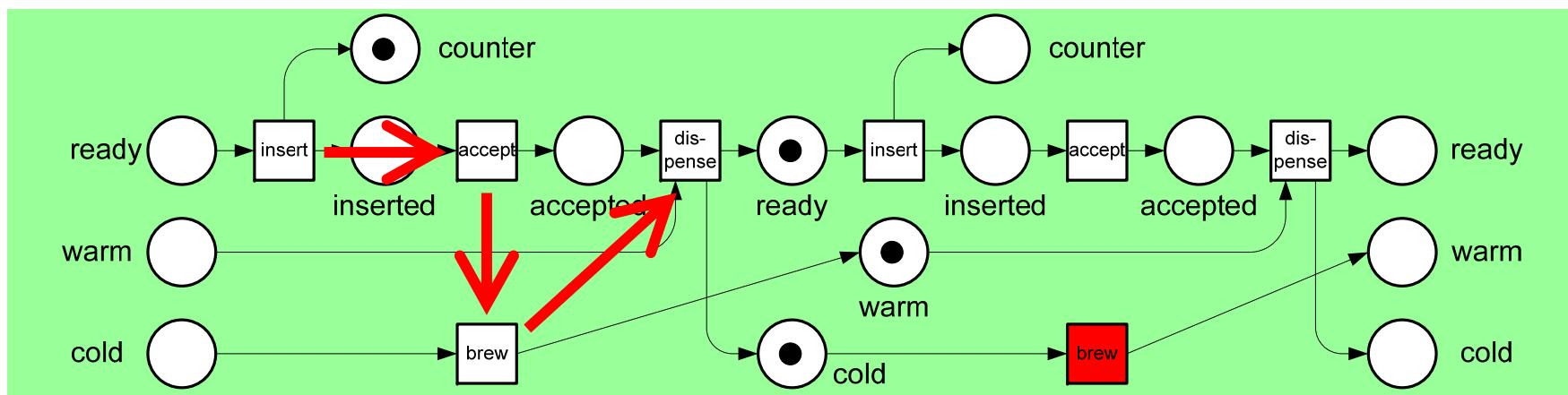
insert, accept, brew, dispense



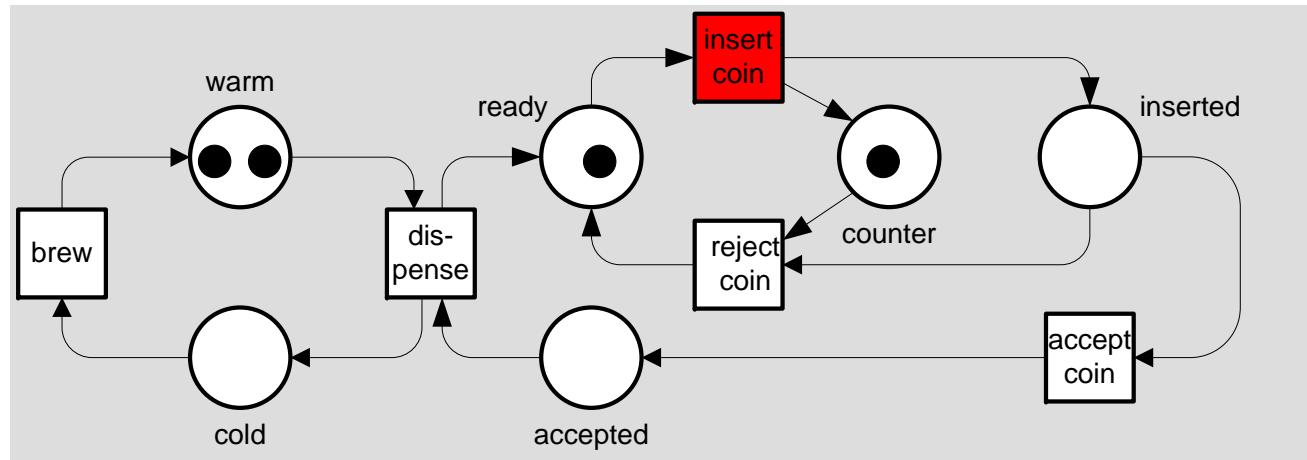
One common occurrence sequence



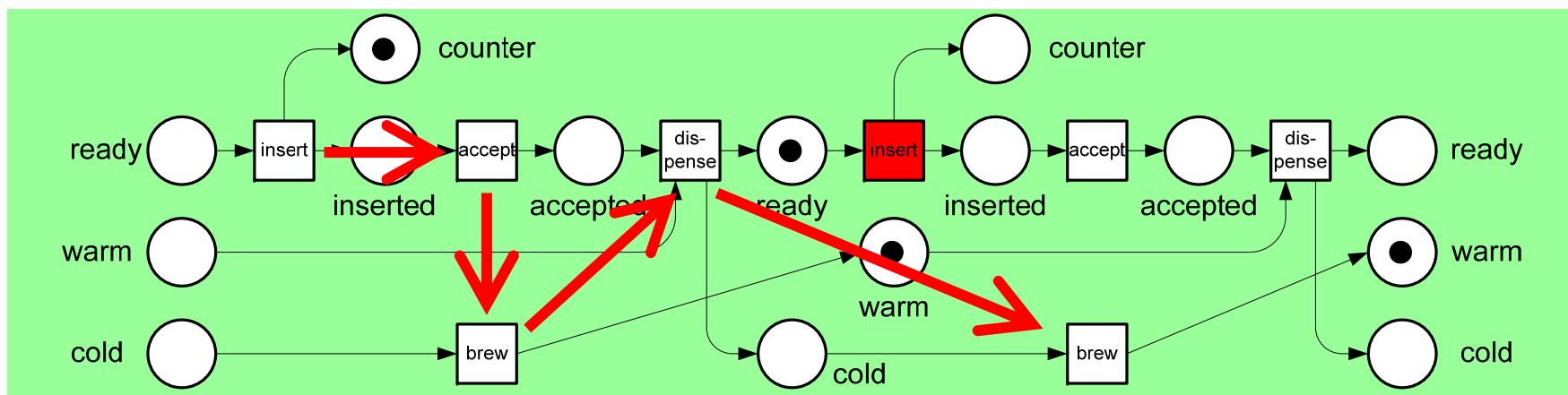
insert, accept, brew, dispense, brew



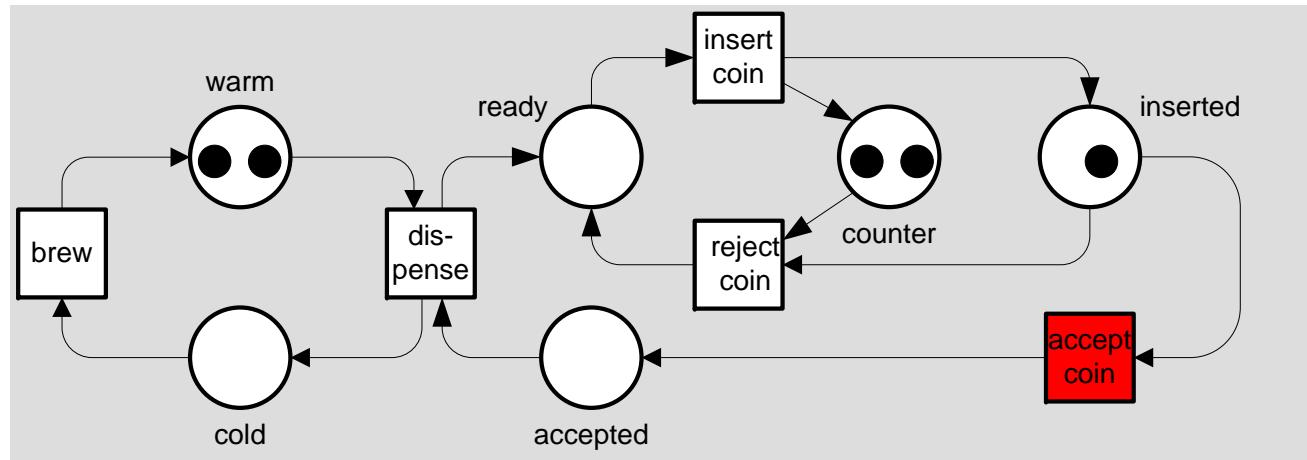
One common occurrence sequence



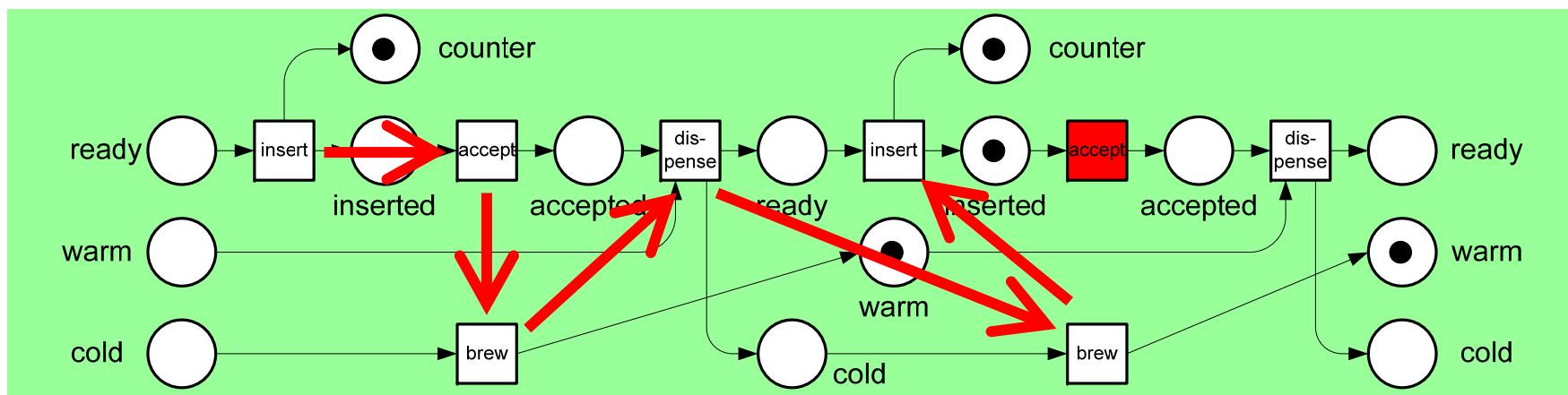
insert, accept, brew, dispense, brew, insert



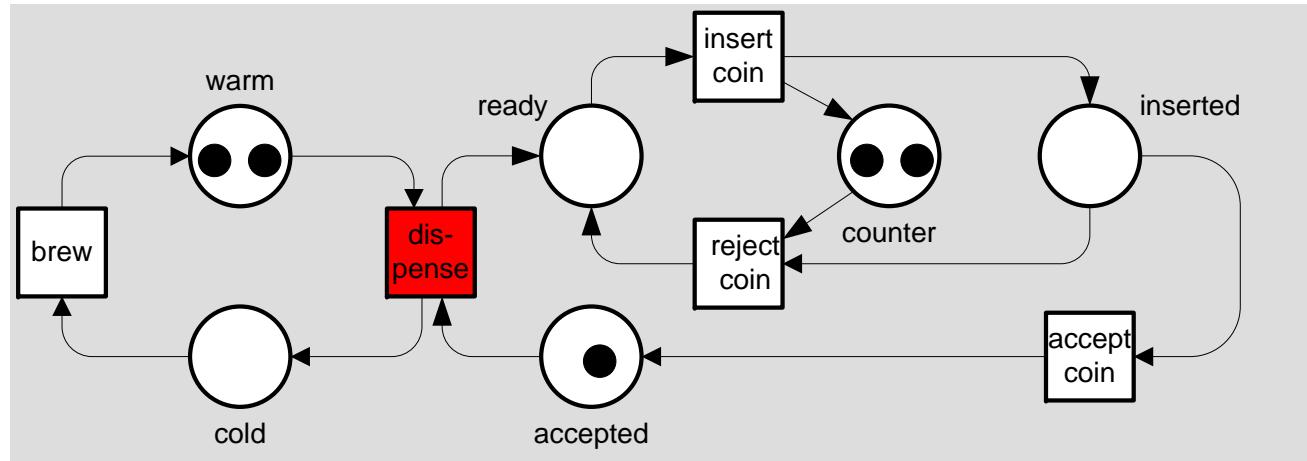
One common occurrence sequence



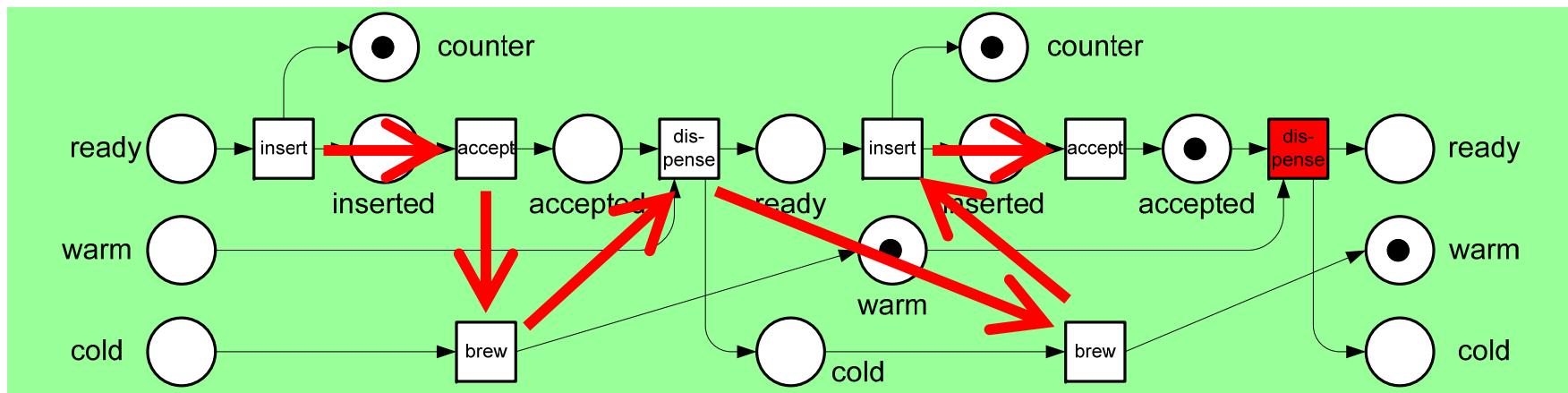
insert, accept, brew, dispense, brew, insert, accept



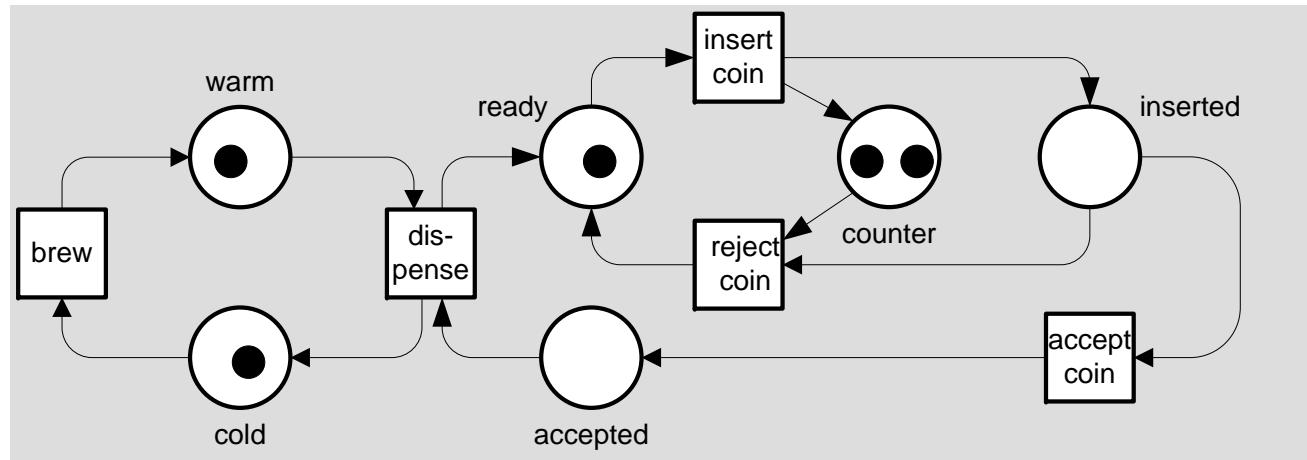
One common occurrence sequence



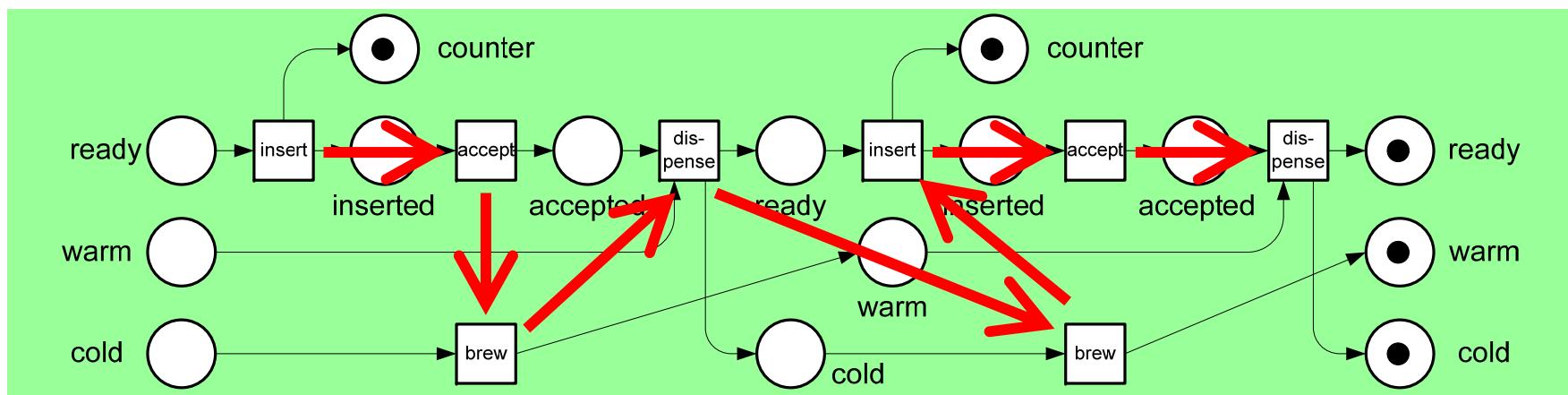
insert, accept, brew, dispense, brew, insert, accept, dispense



One common occurrence sequence



insert, accept, brew, dispense, brew, insert, accept, dispense

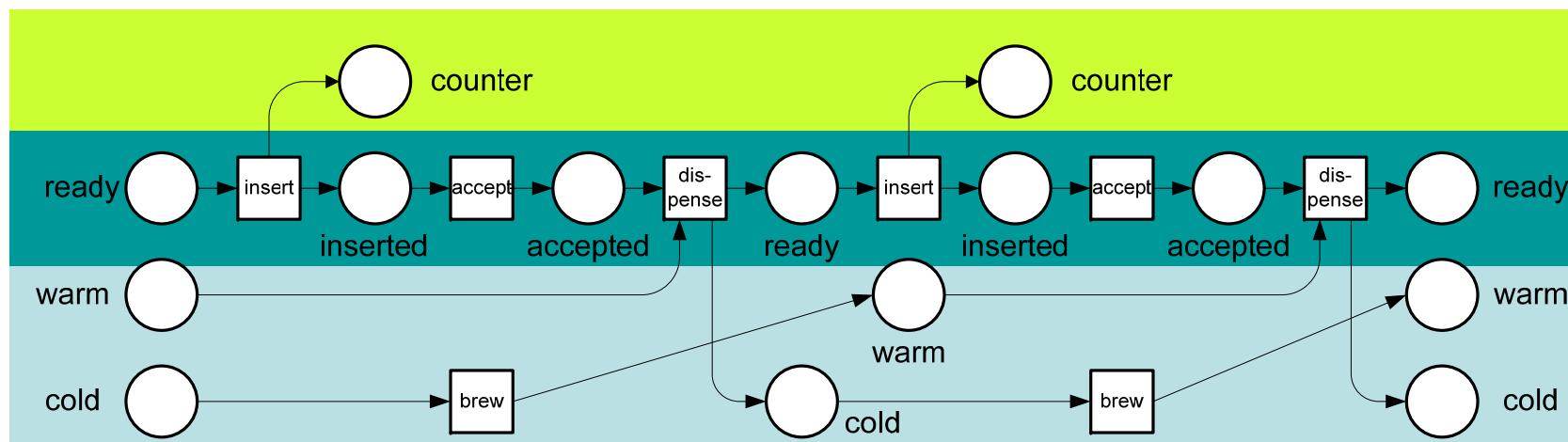
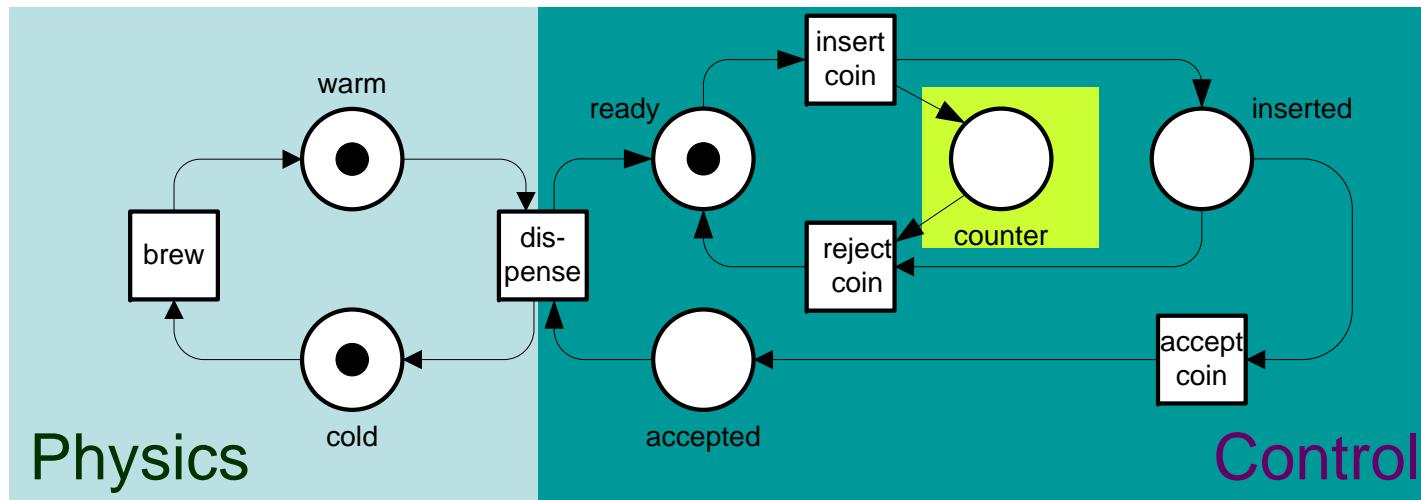


a linearization of the partial order of events

Advantage 2: Expressiveness

components of models, token flow etc.
can easily be identified in process nets

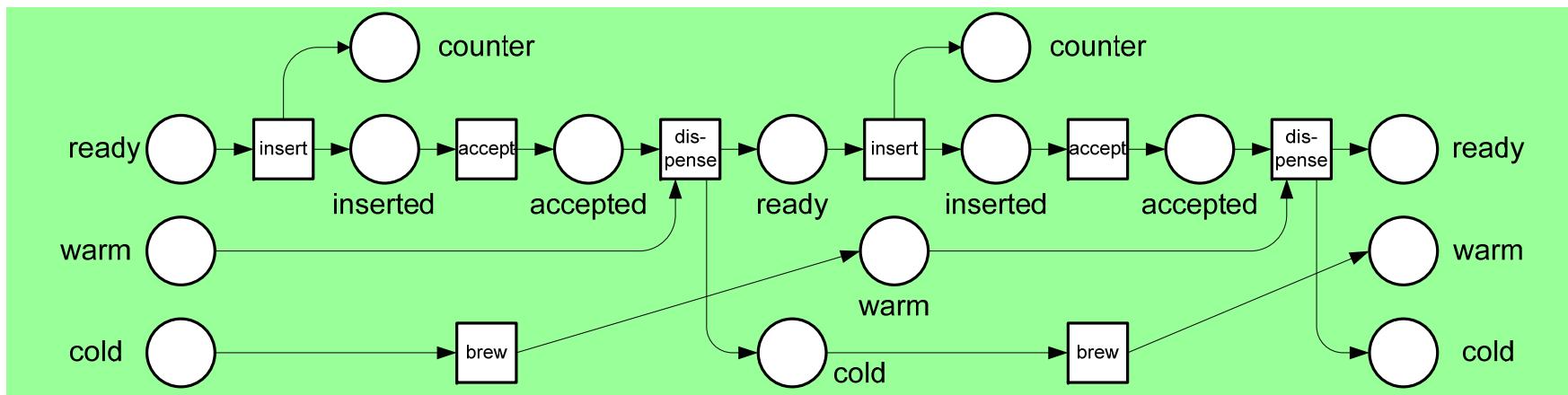
Advantage 2: Expressiveness



Advantage 3: Expressiveness (again)

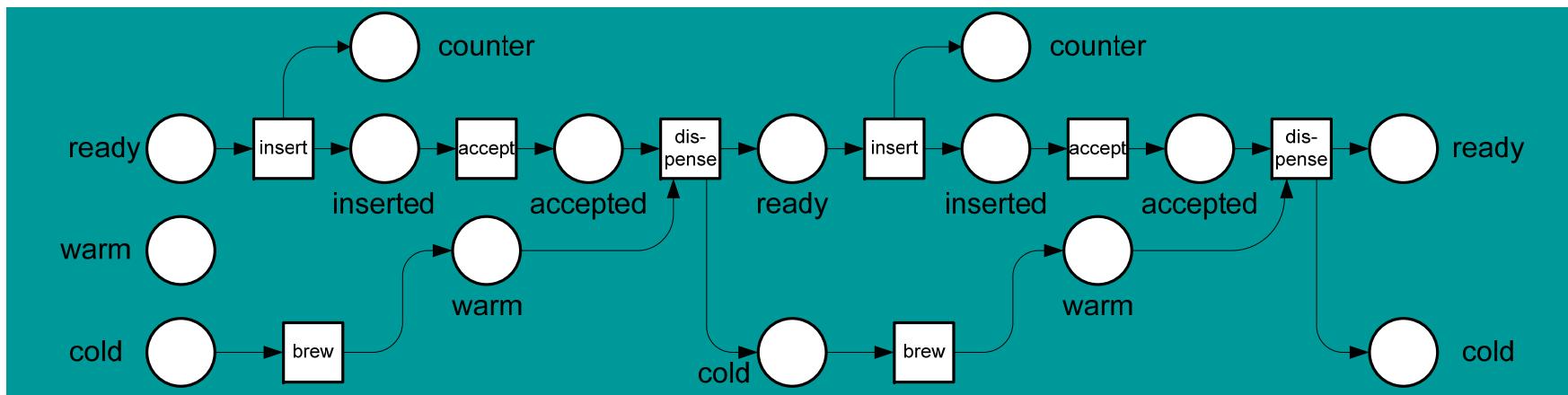
causal relationships are explicitly represented
(which is not the case for occurrence sequences
of non-safe place/transition nets)

Advantage 3: Expressiveness (again)



a common occurrence sequence of both process nets:

insert, accept, brew, dispense, brew, insert, accept, dispense



Validating Requirements

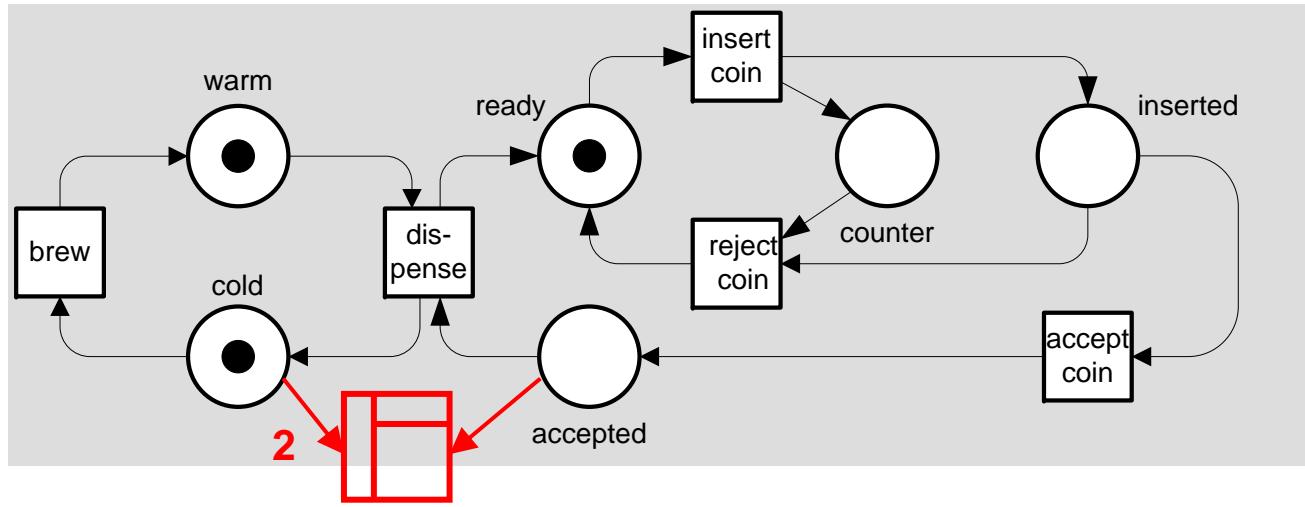
What type of requirements?

- linear-time properties that can be interpreted on single process nets

Two examples:

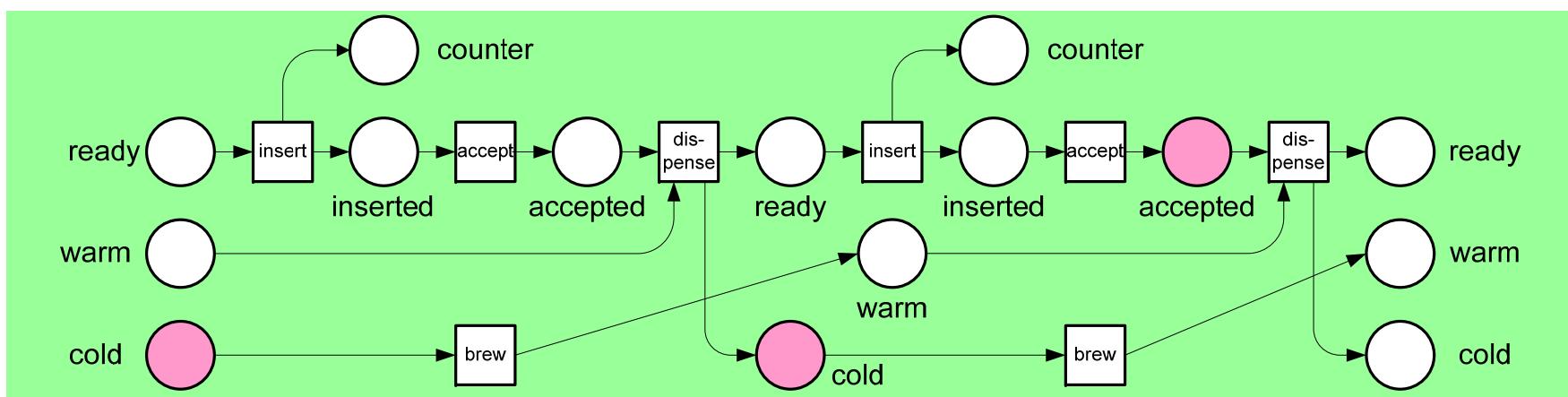
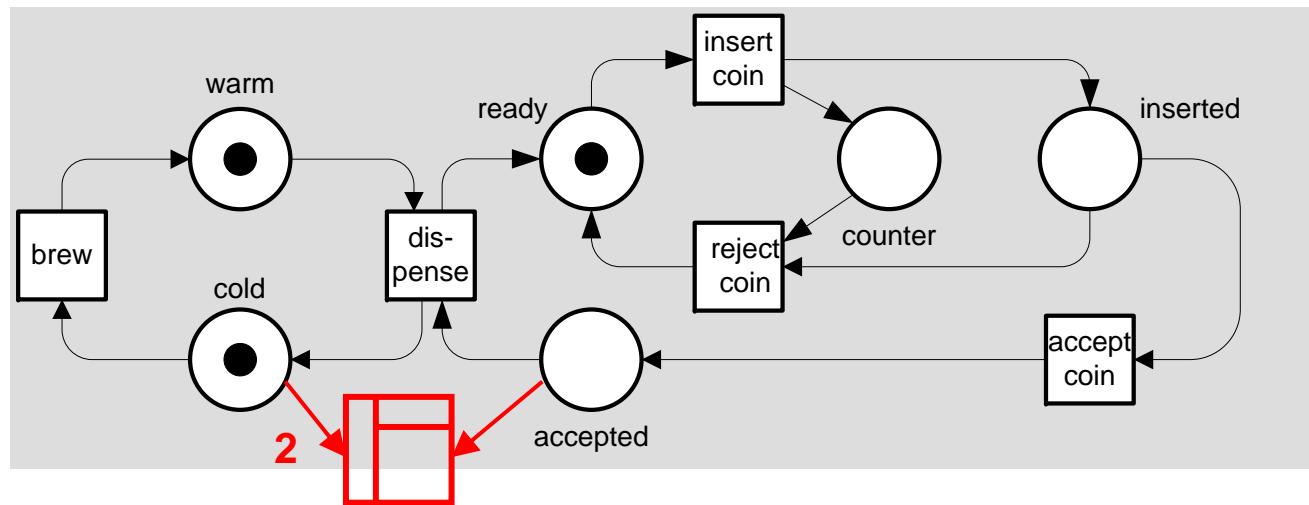
- facts representing invariant properties
- goals representing “leads-to”-properties

A Fact



identify processes with a reachable marking
marking **cold** twice and marking **accepted**

A Fact



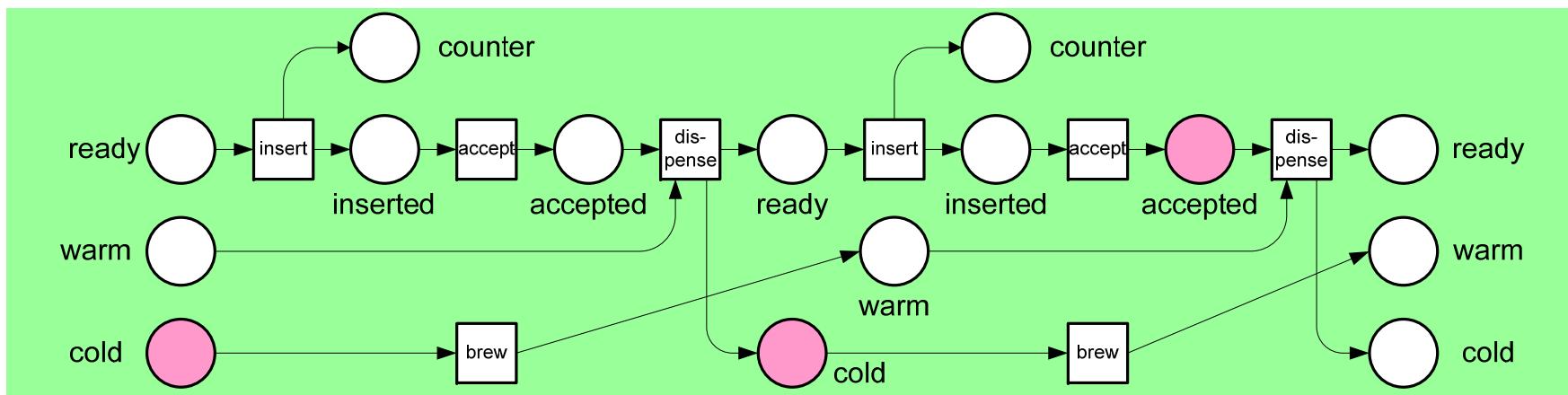
A Fact

A co-set of appropriately marked places of the process...

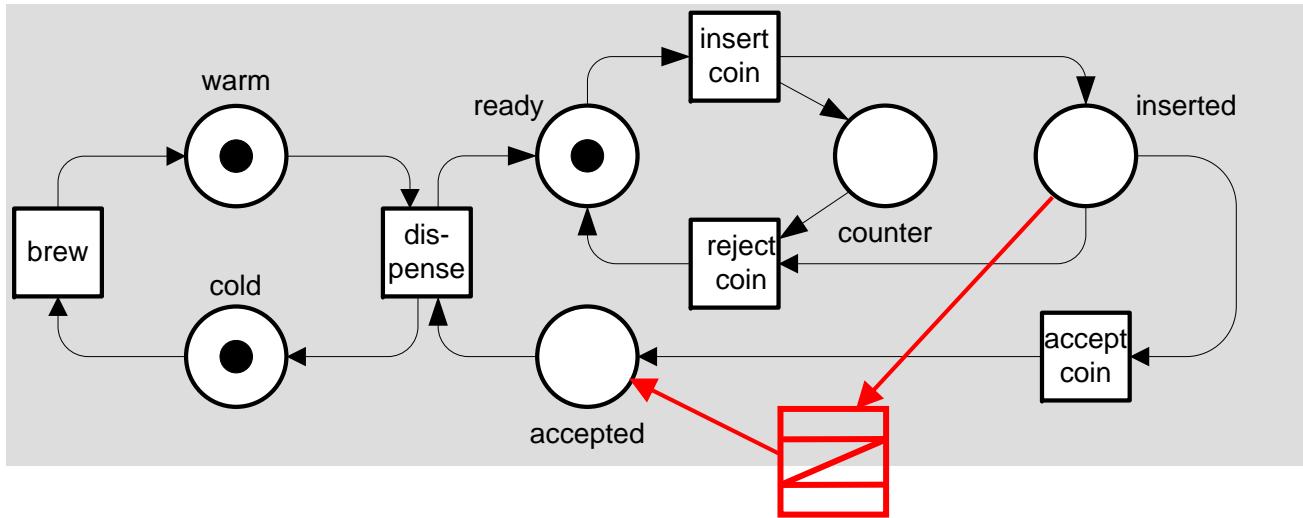
... is included in a cut

... which corresponds to a reachable marking of the process

... which corresponds to a reachable marking of the net.



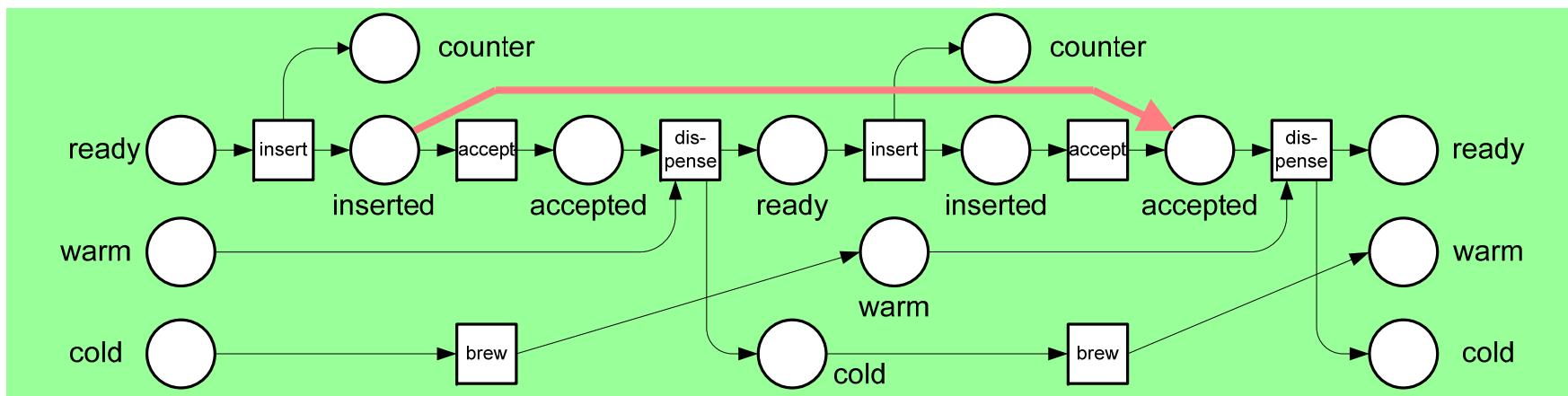
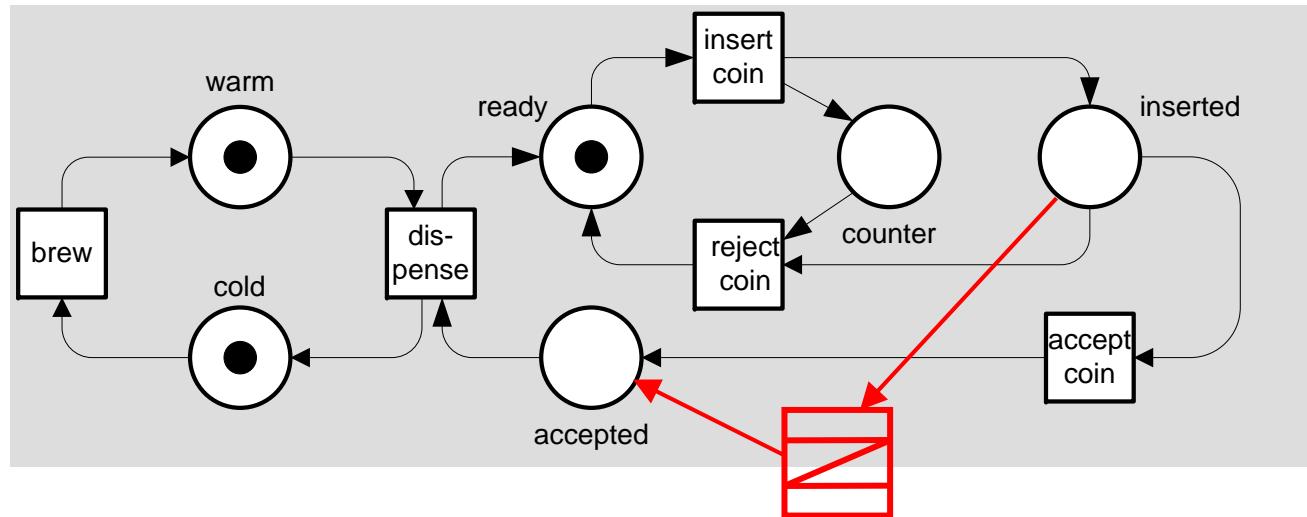
A Goal



identify processes such that:

If there is a token on *inserted* then eventually
there will be a token on *accepted*

A Goal

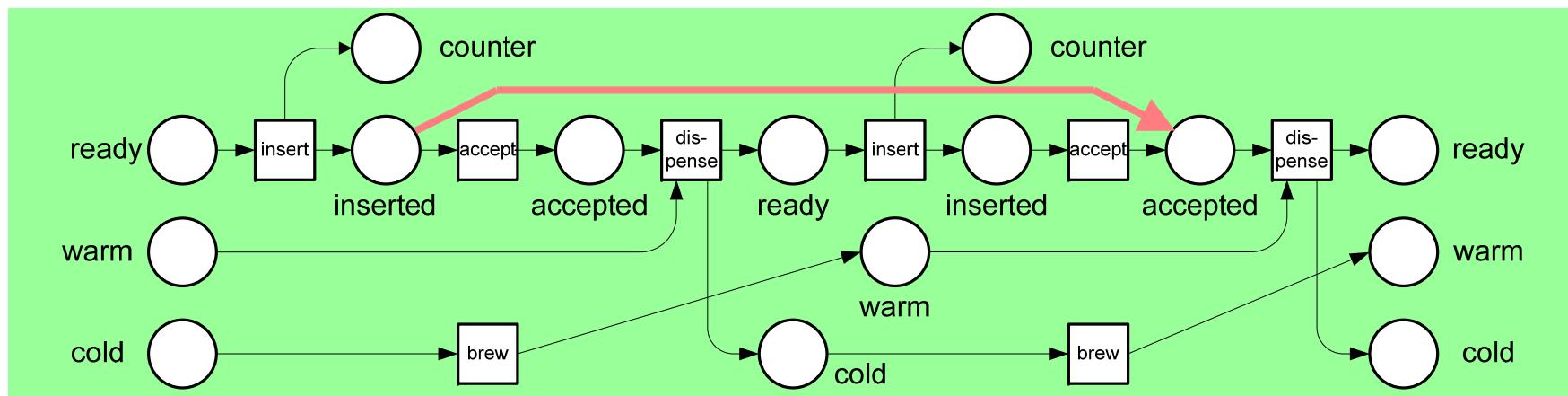


A Goal

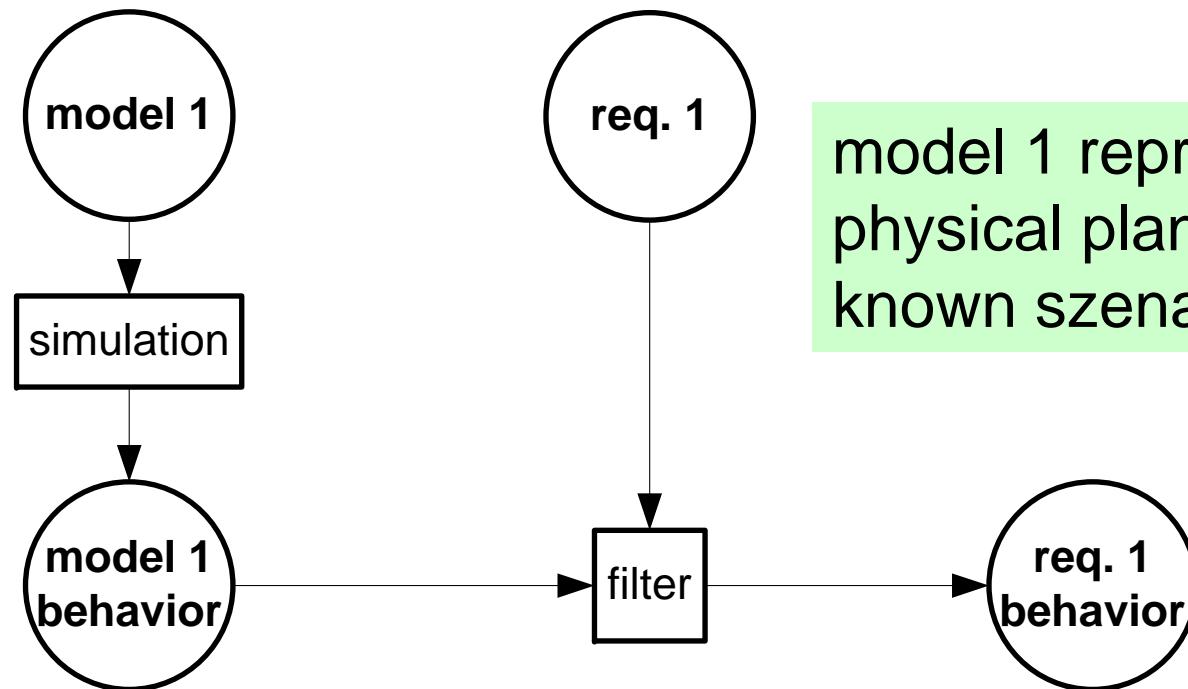
accepted causally depends on **inserted**

... hence from each marking of the process
that marks the condition **inserted**
a marking will be reached that marks condition **accepted**.

So for this run, the place
accepted will eventually be marked after **inserted**.

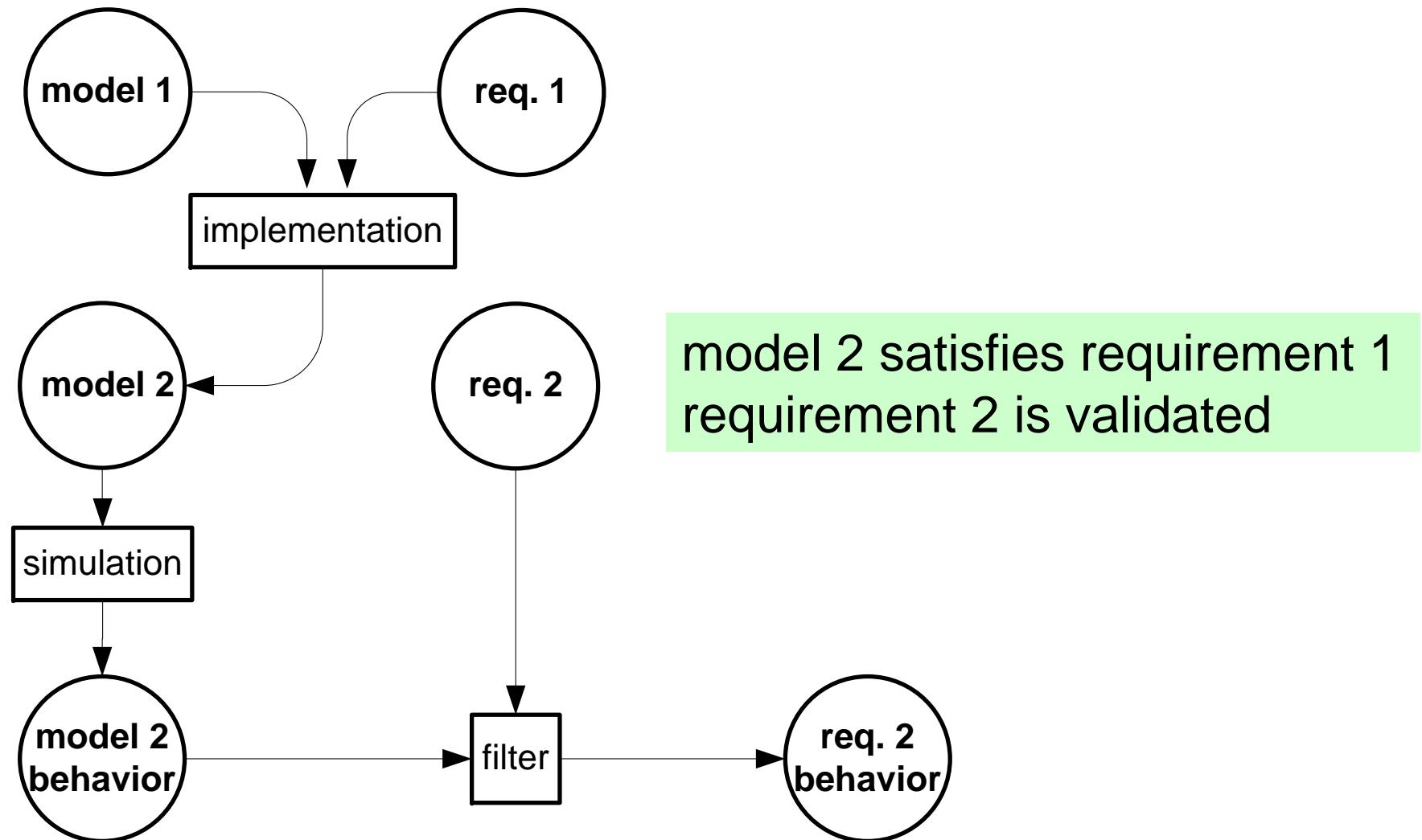


Stepwise Validation of Requirements

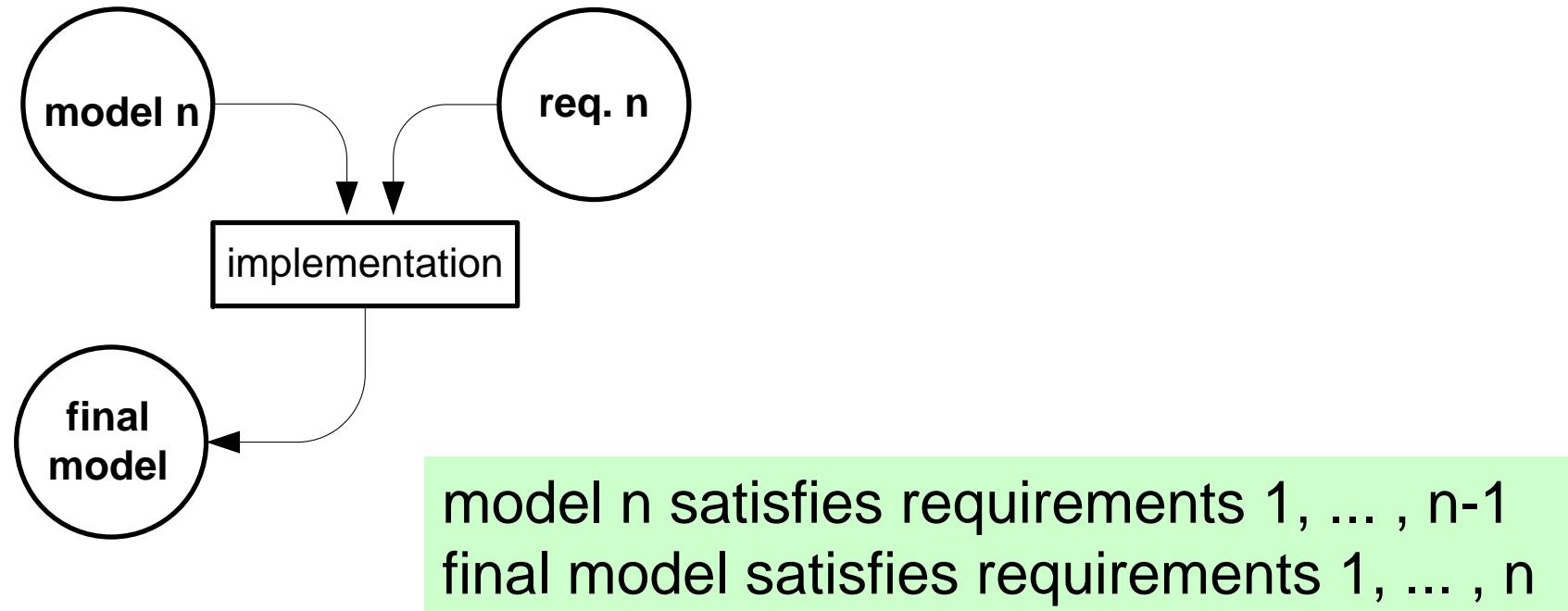


model 1 represents the physical plant,
known scenarios etc.

Stepwise Validation of Requirements



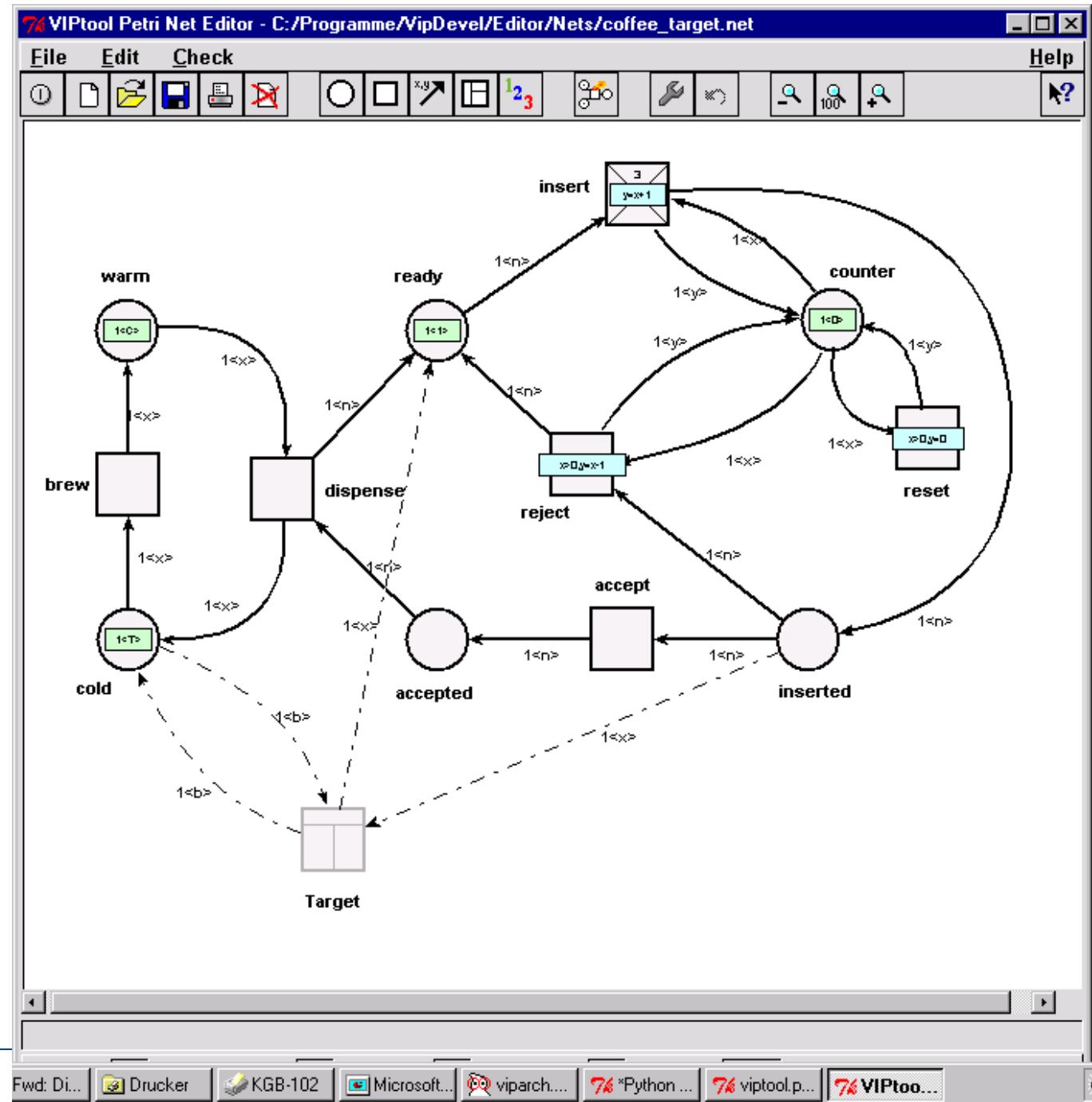
Stepwise Validation of Requirements



**This approach only works in general
if all requirements restrict behavior**

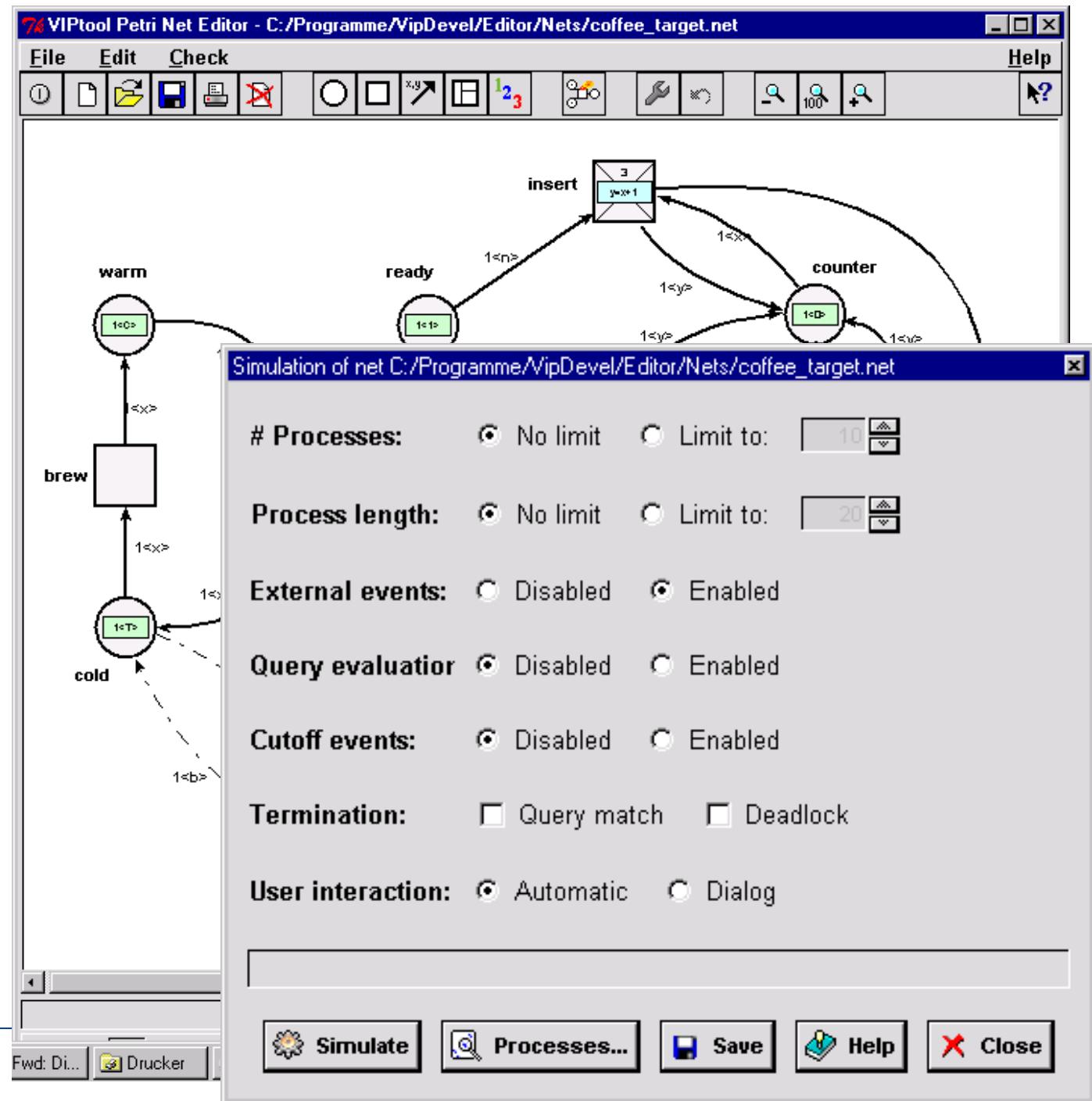
VipTool

simulation
generates
causal nets



VipTool

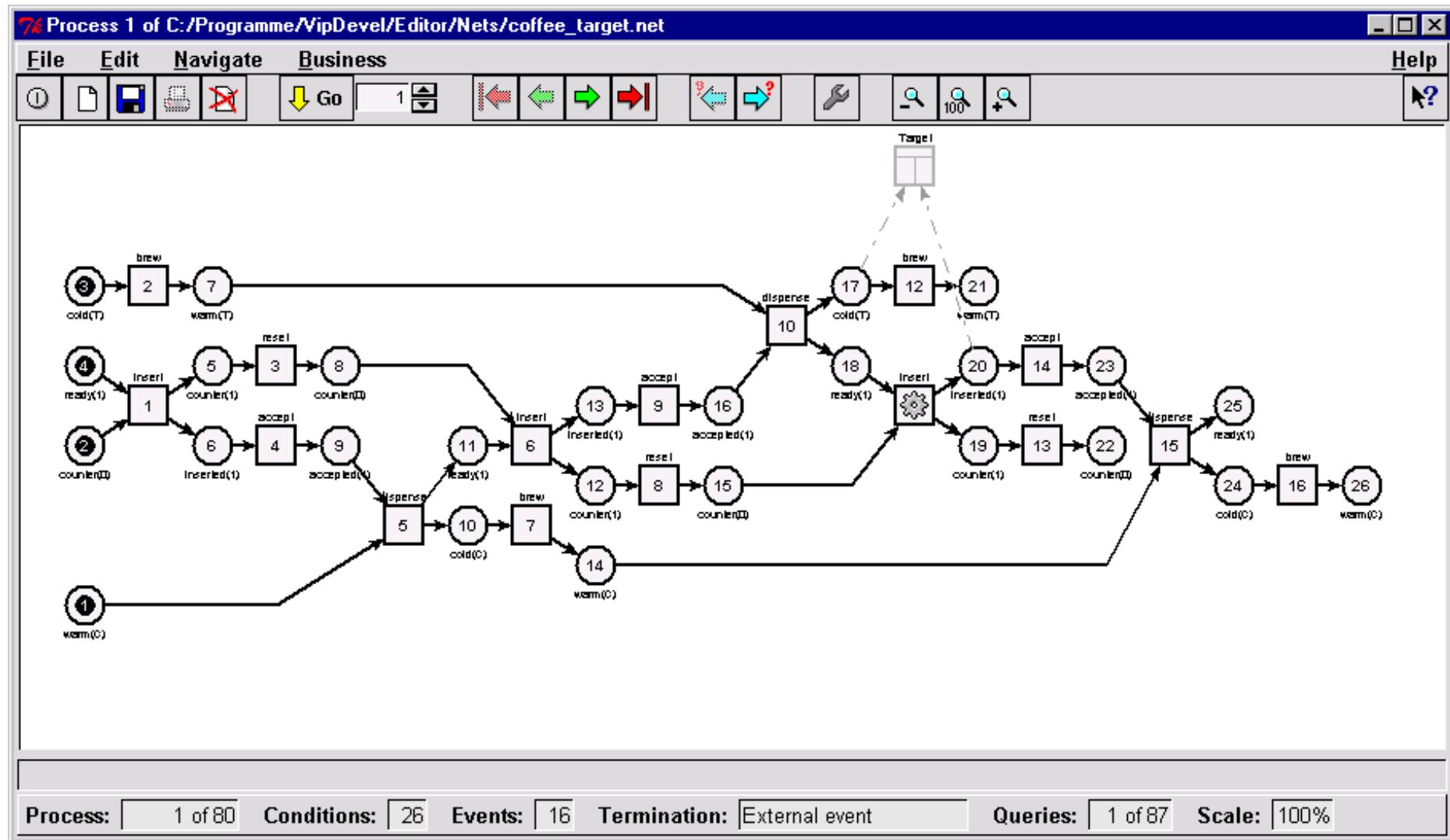
control of the simulation



VipTool



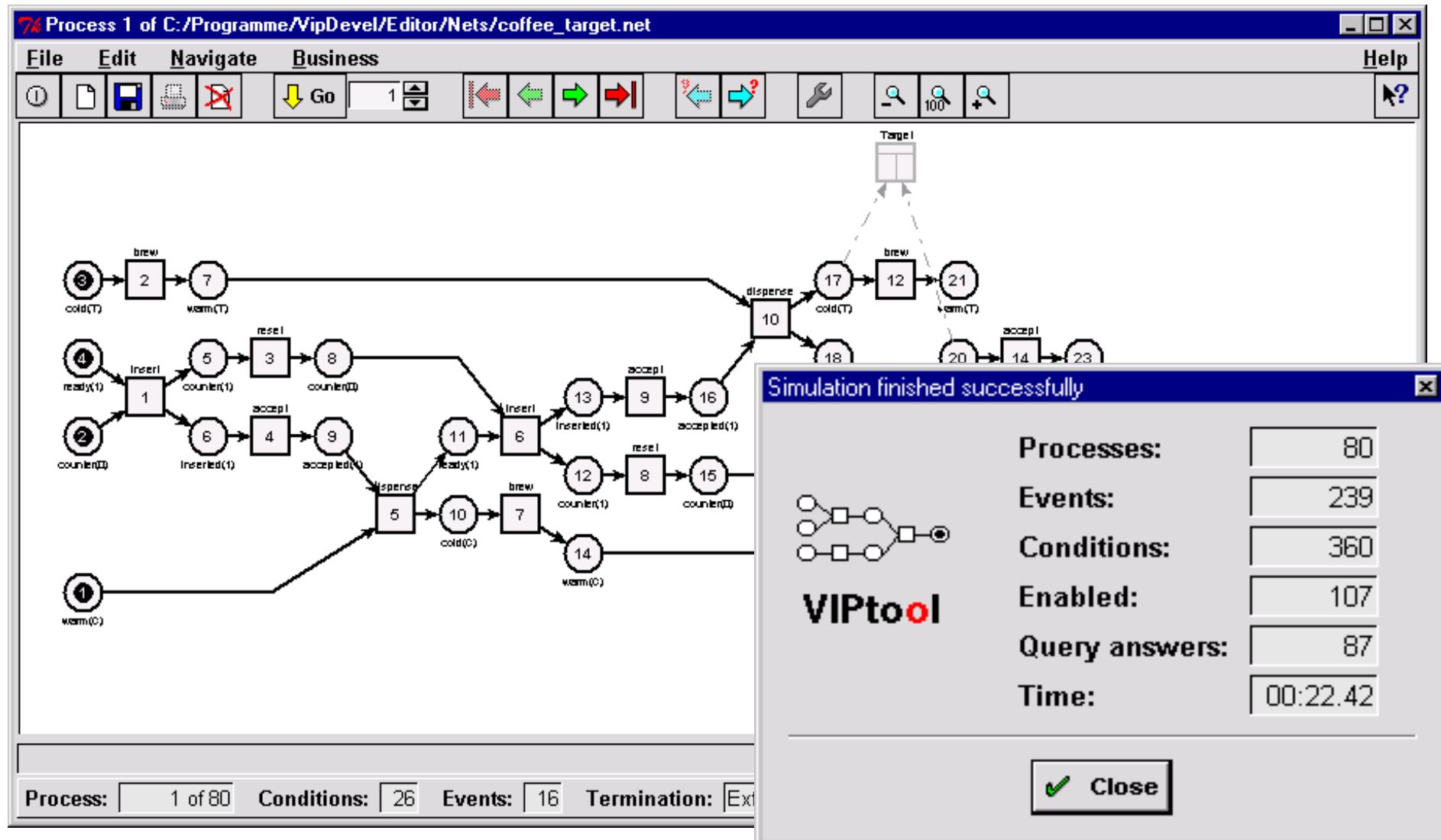
a generated and visualized run



VipTool



analysis of runs

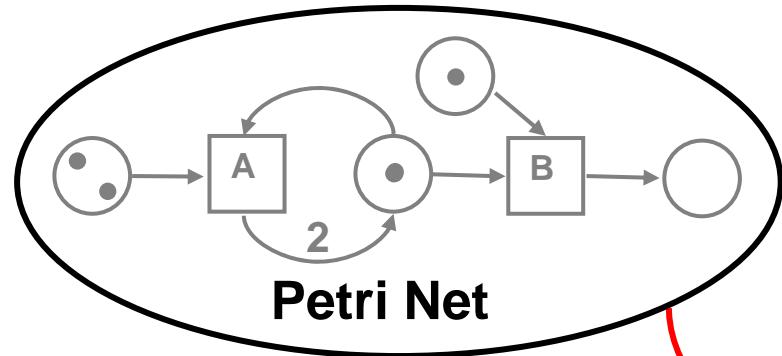


Synthesis

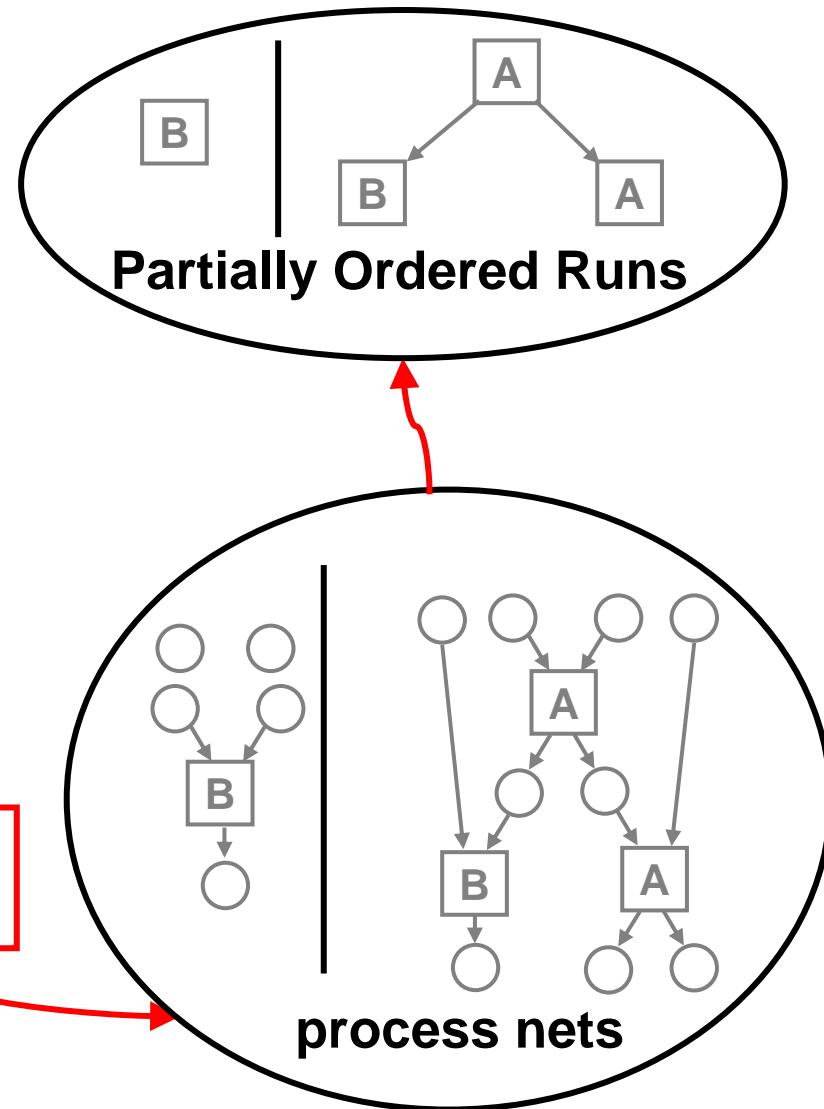
means generation of nets from runs.

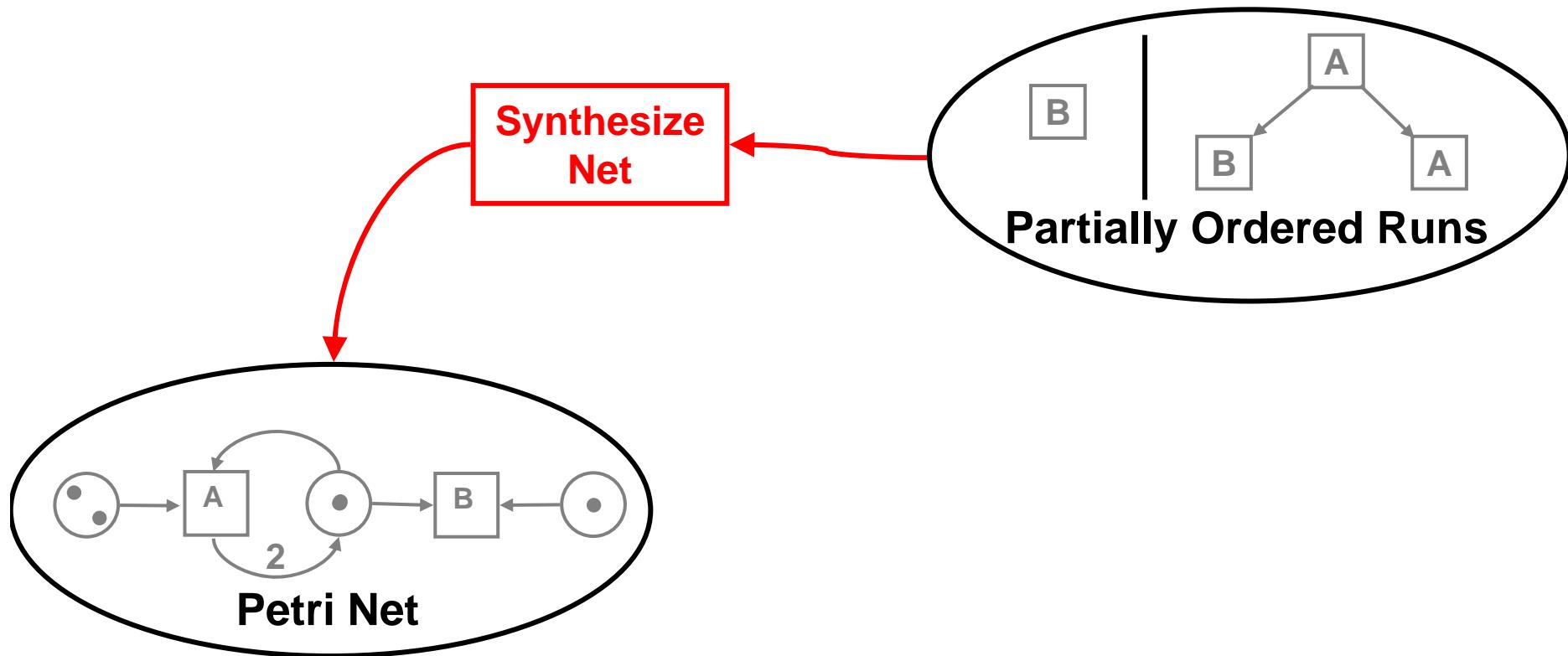
from

- **sequential runs** (occurrence sequences),
 ⇒ well-known region theory
- **non-sequential, partially ordered runs**
 ⇒ the VIP-approach



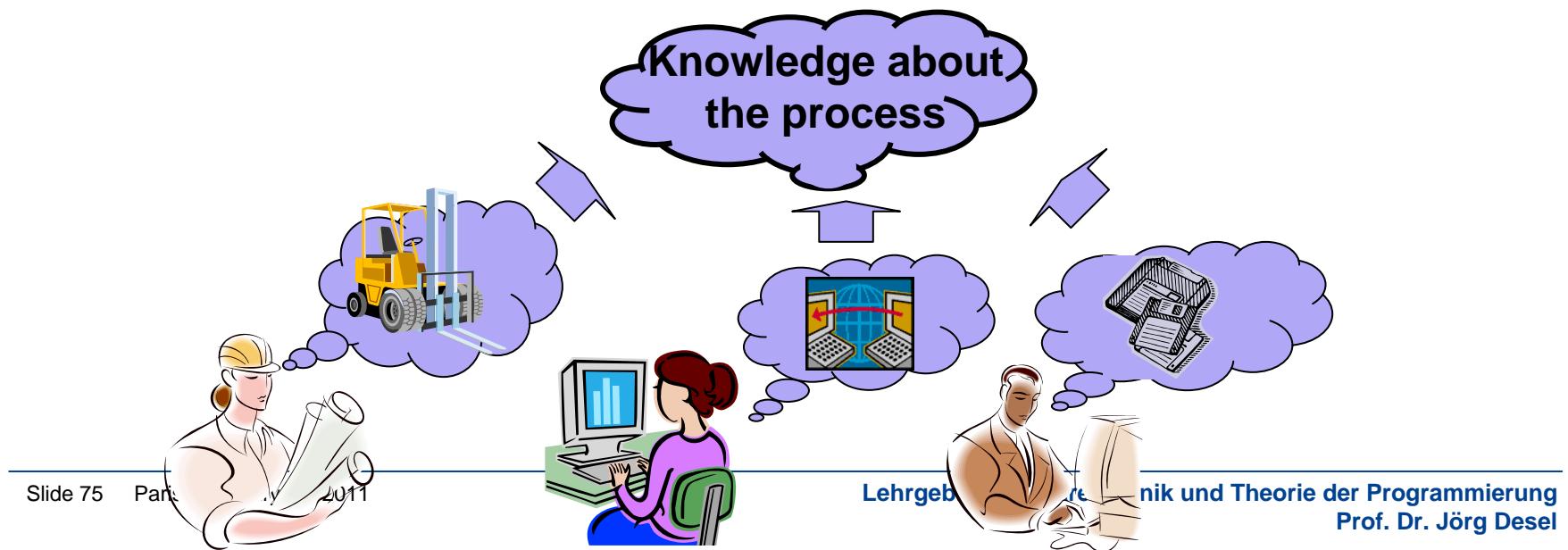
**Unfold to
Behaviour**

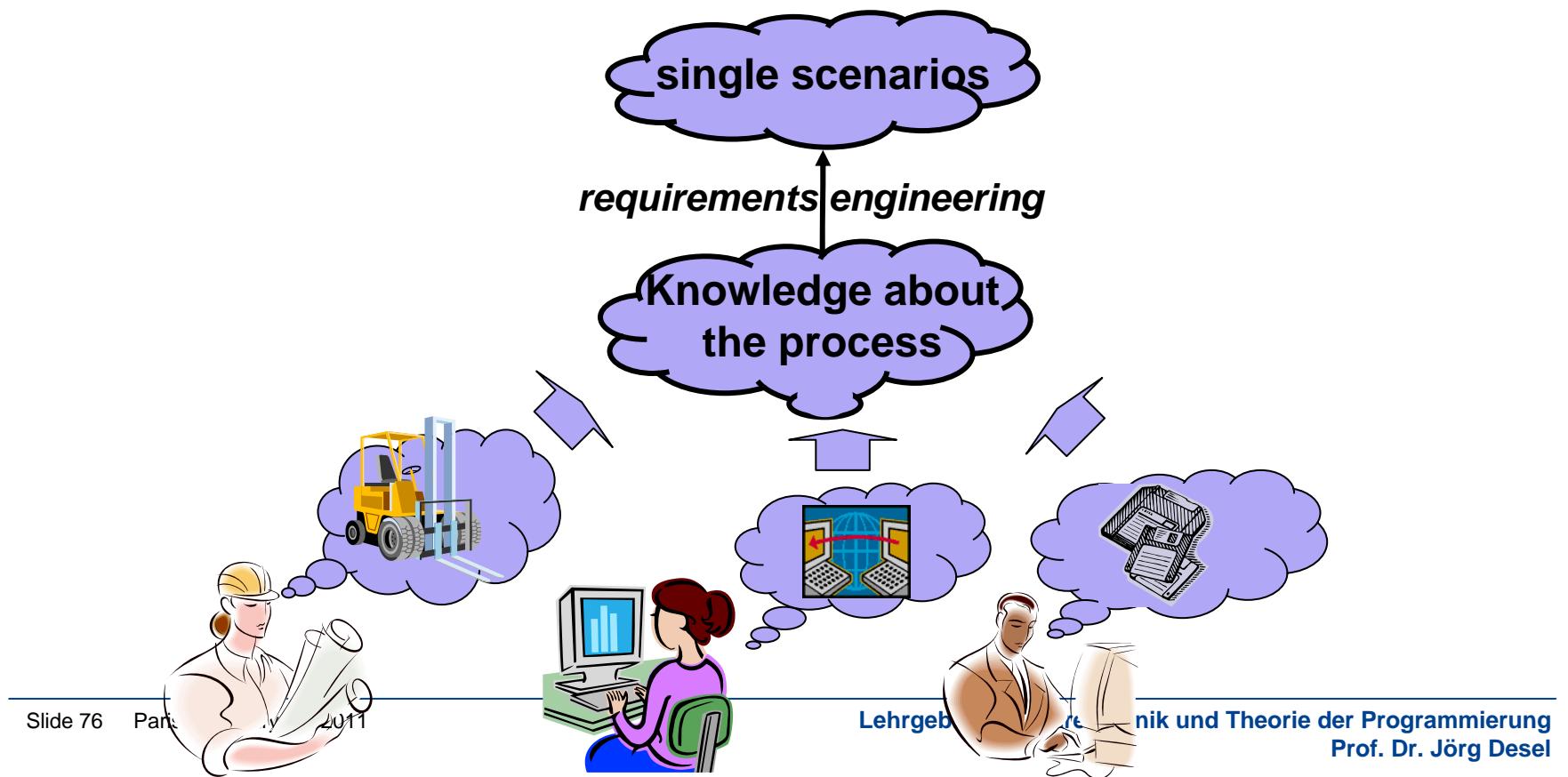


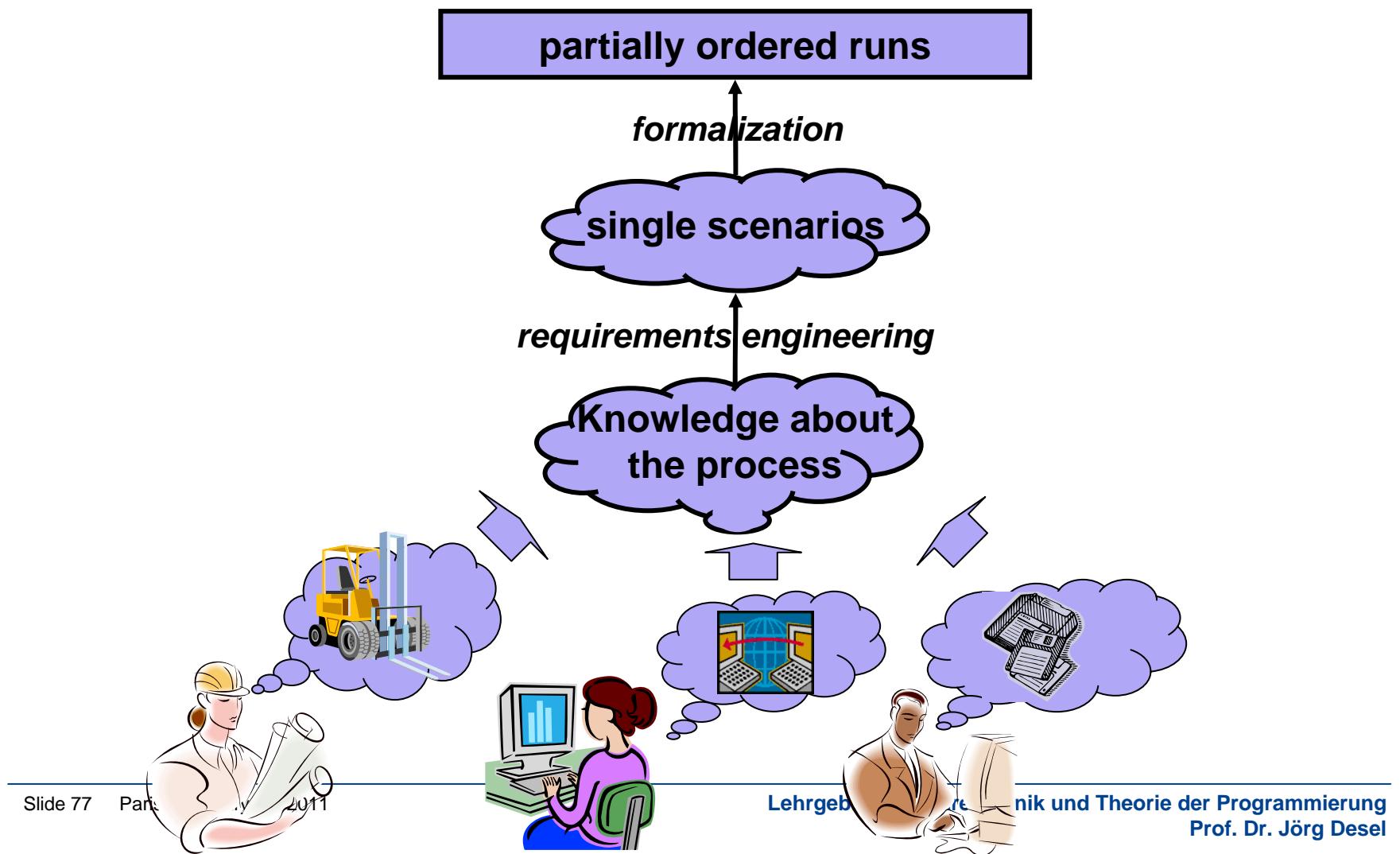


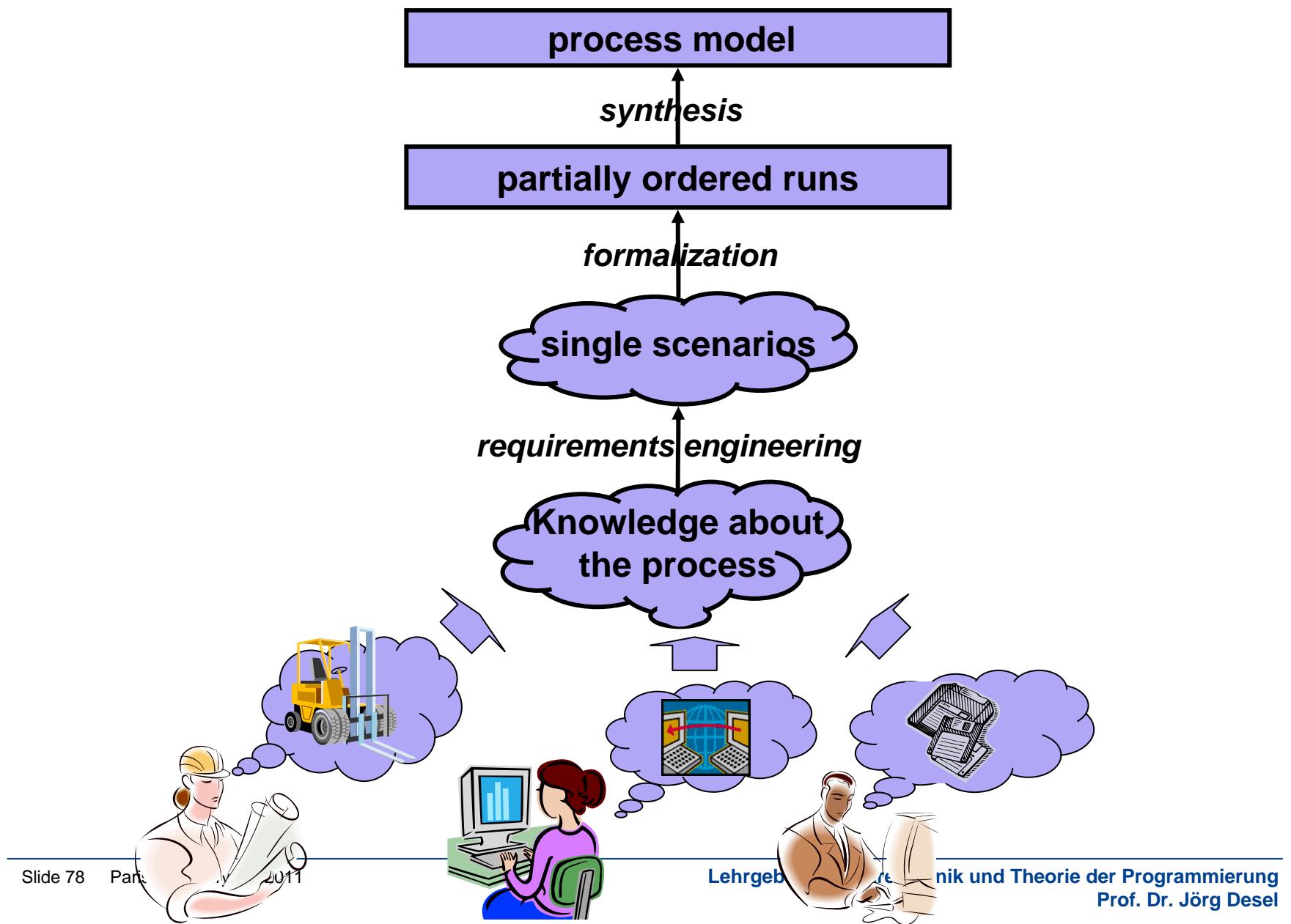
Initial Situation:

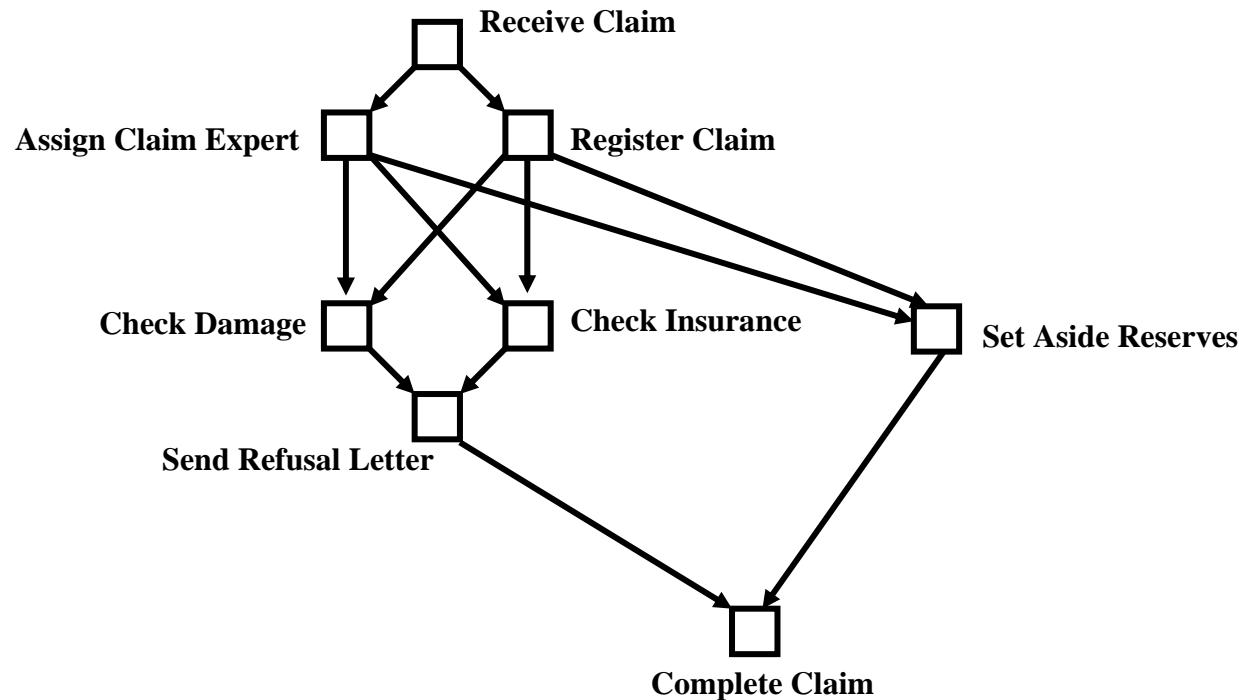
Knowledge about a process is distributed in several peoples' mind in an informal environment



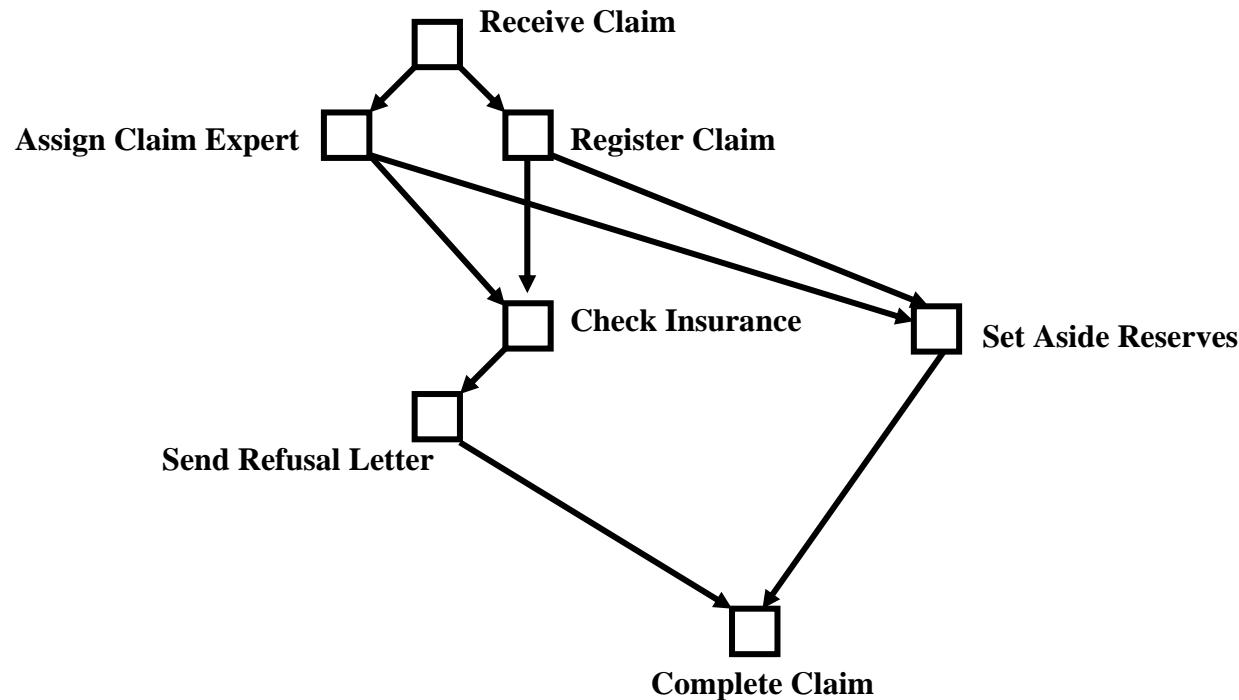




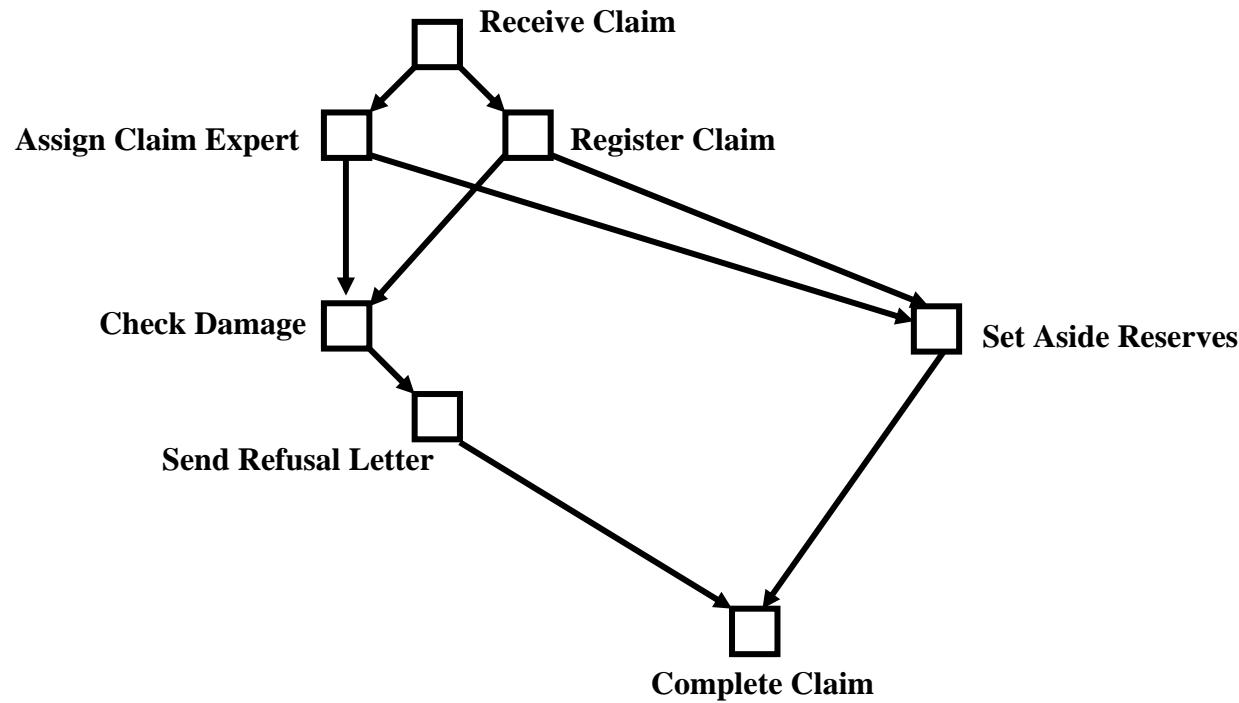




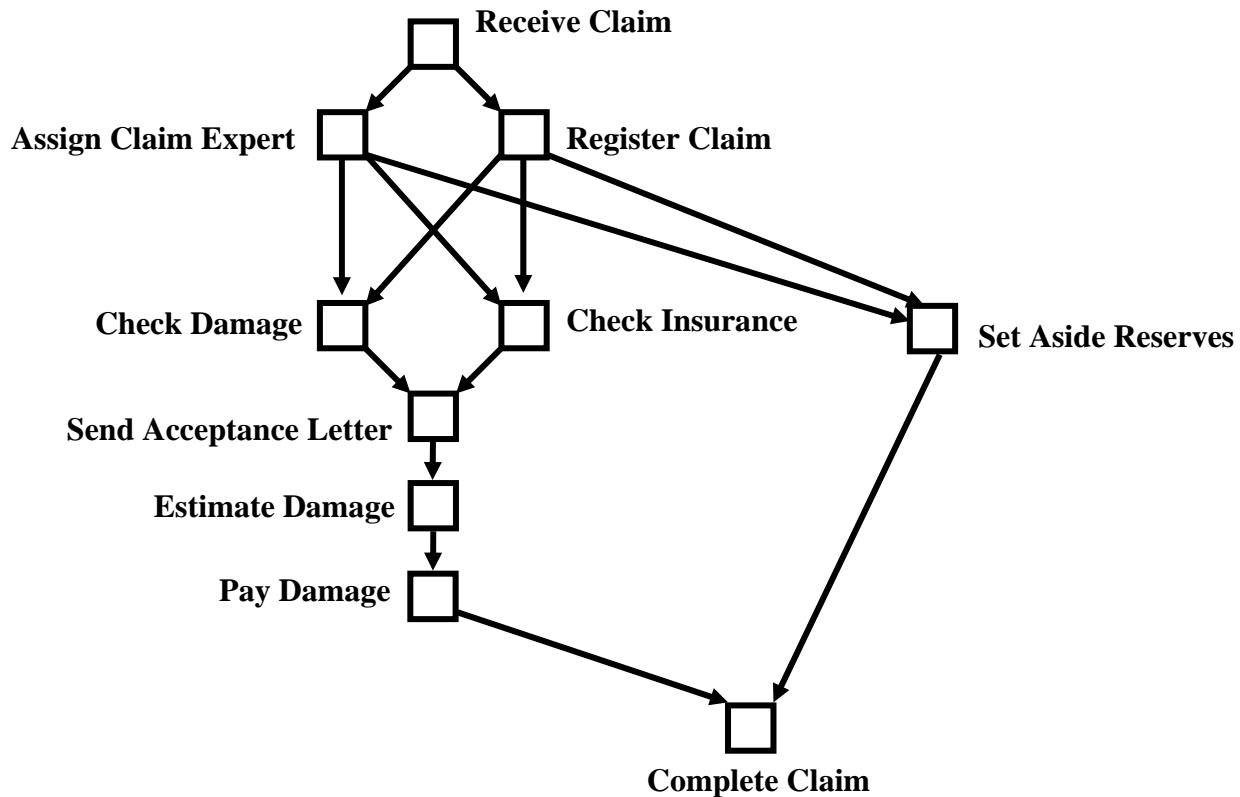
partially ordered runs



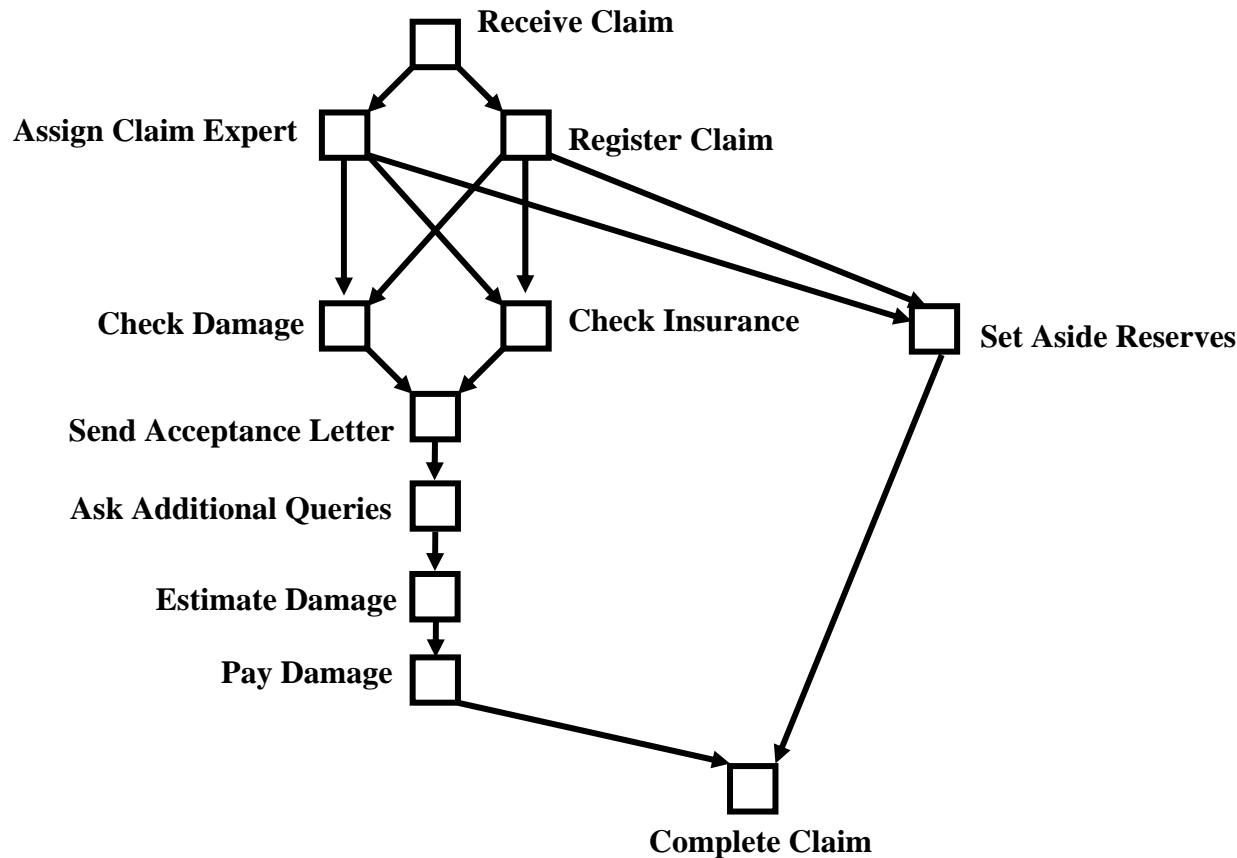
partially ordered runs



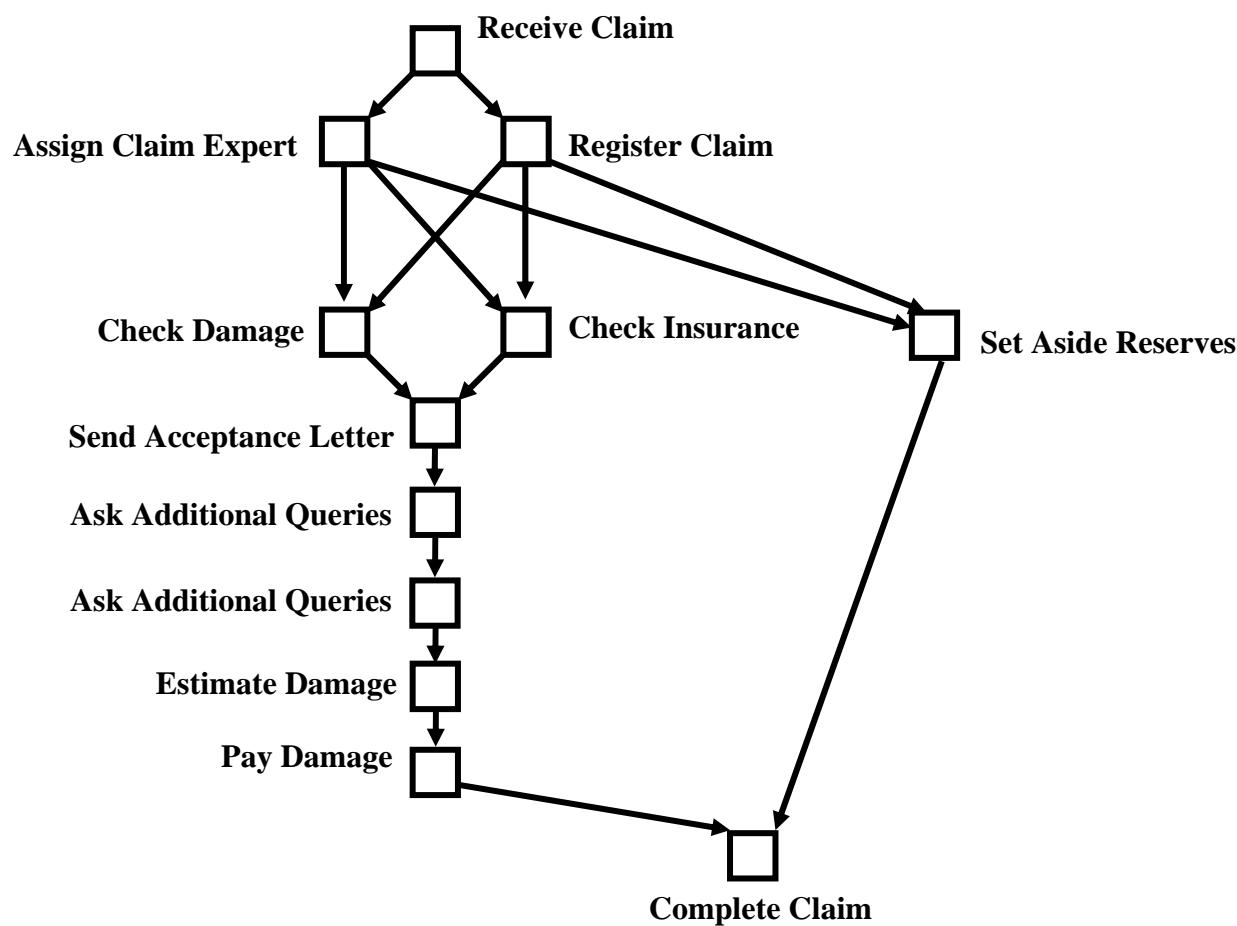
partially ordered runs



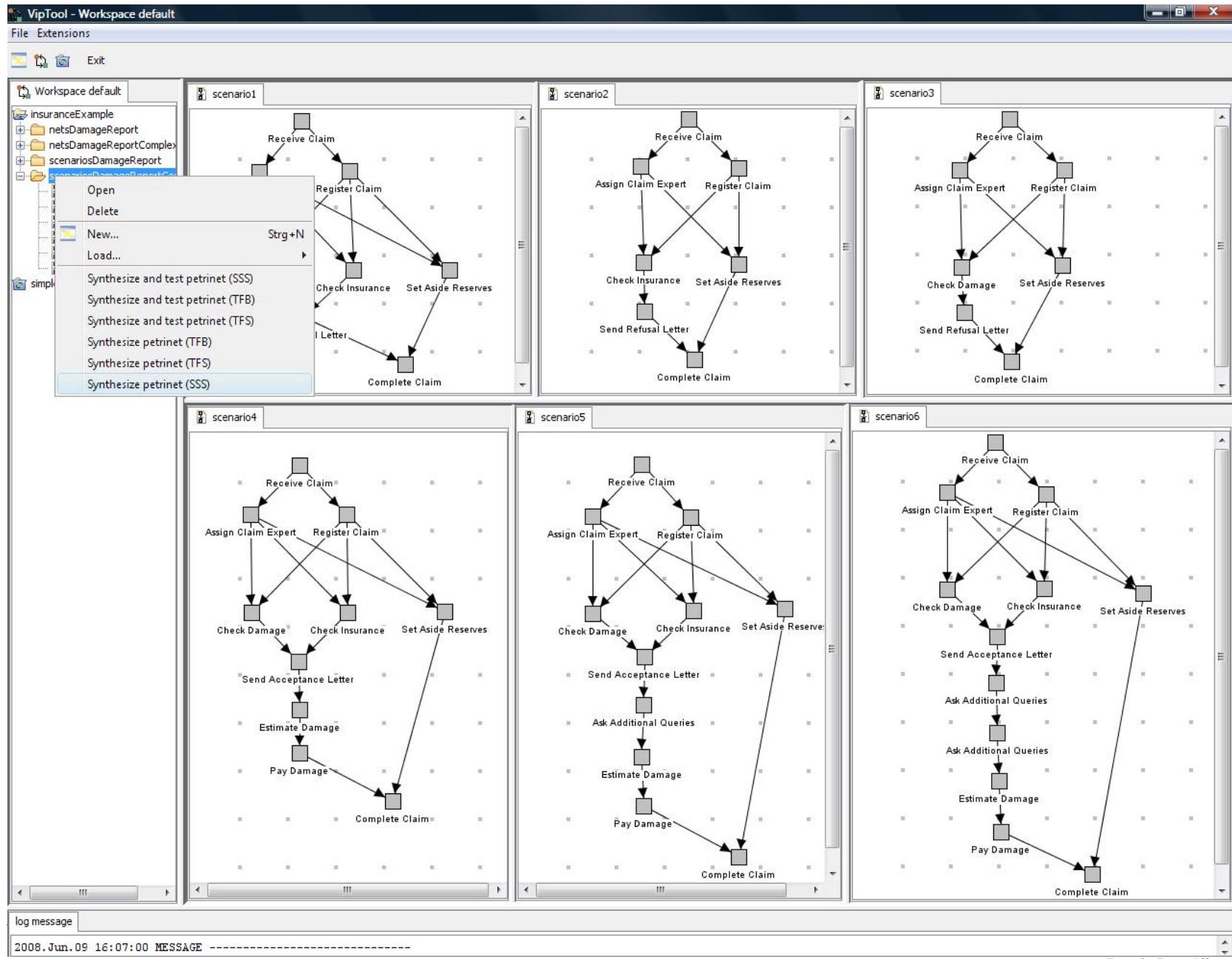
partially ordered runs

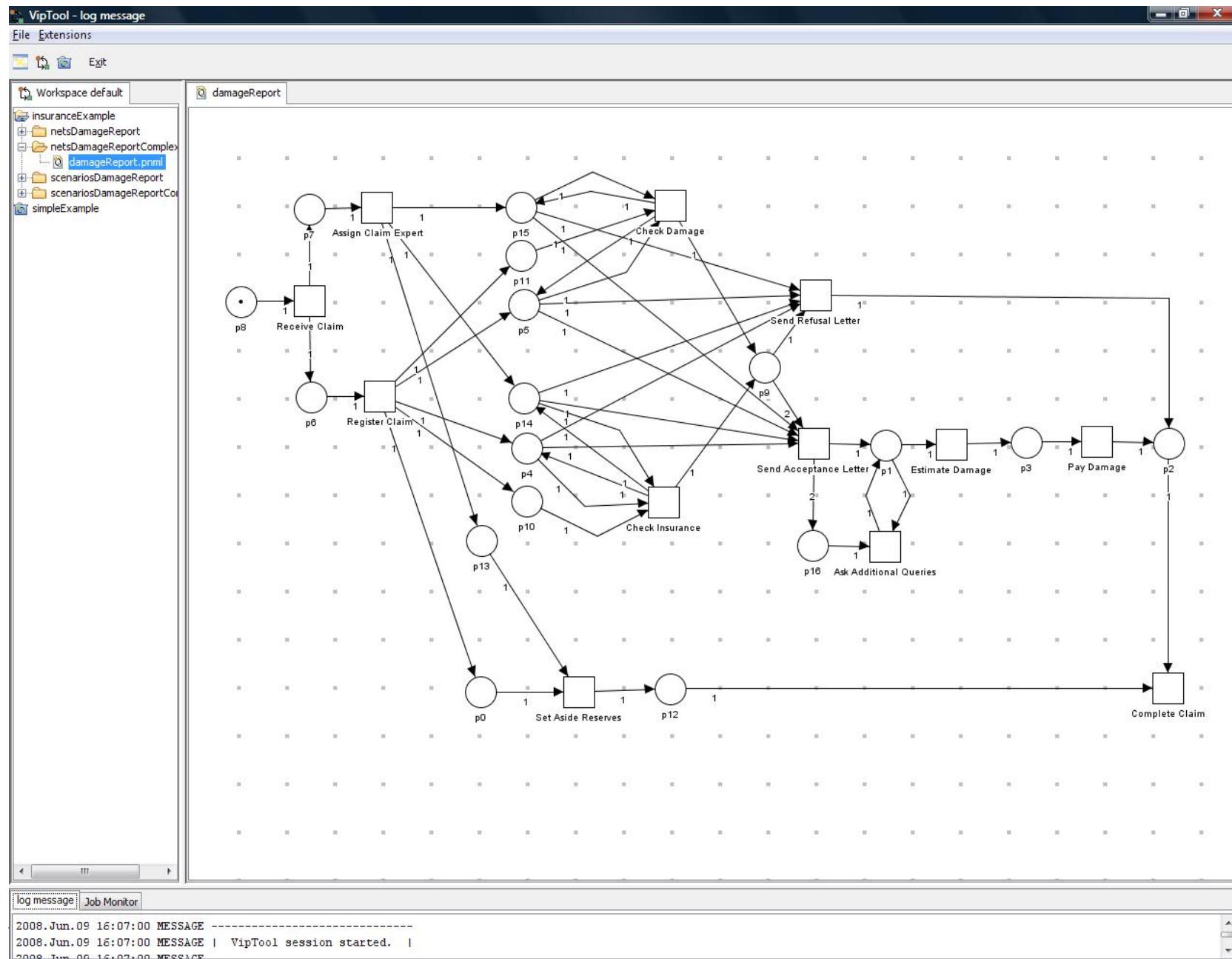


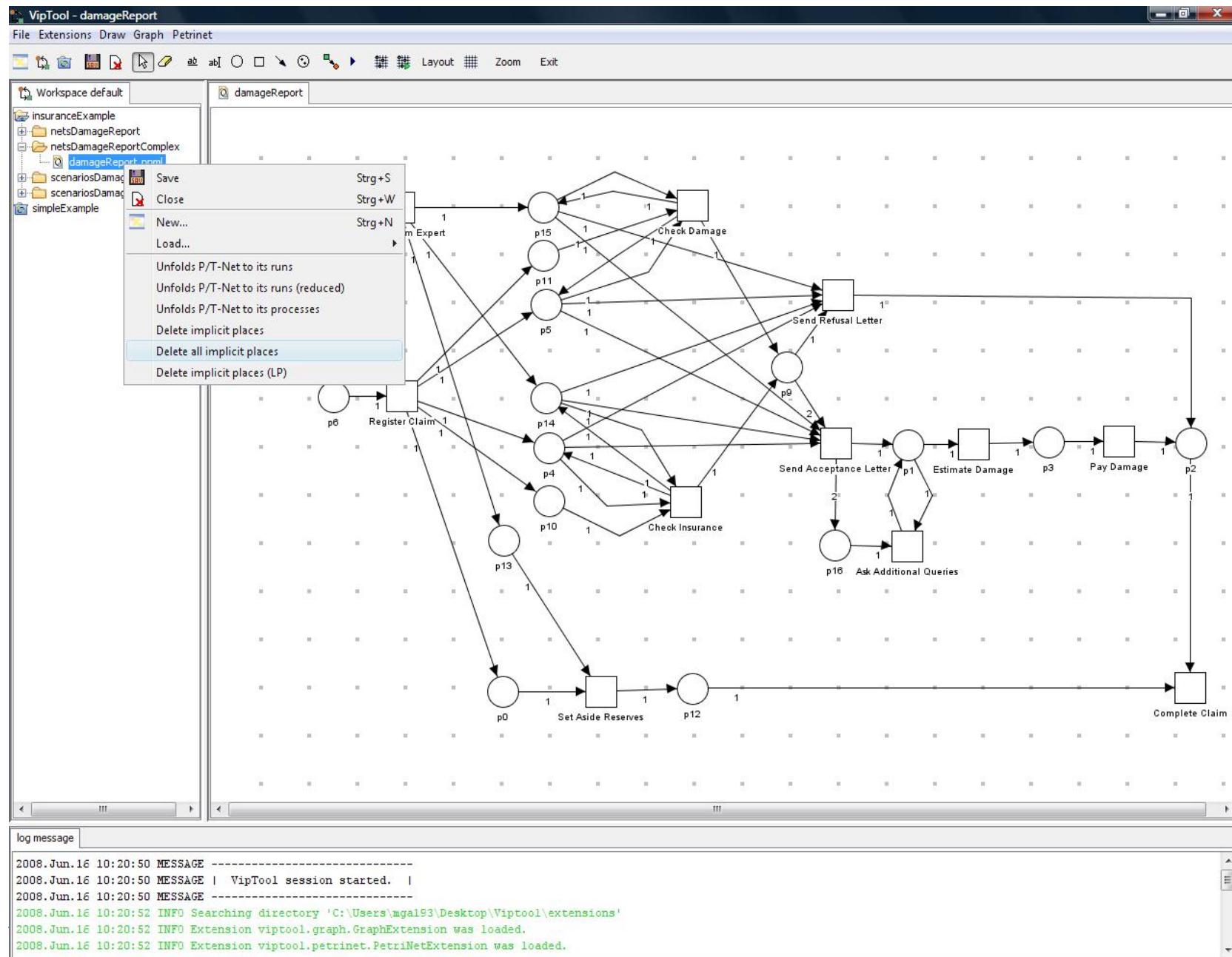
partially ordered runs

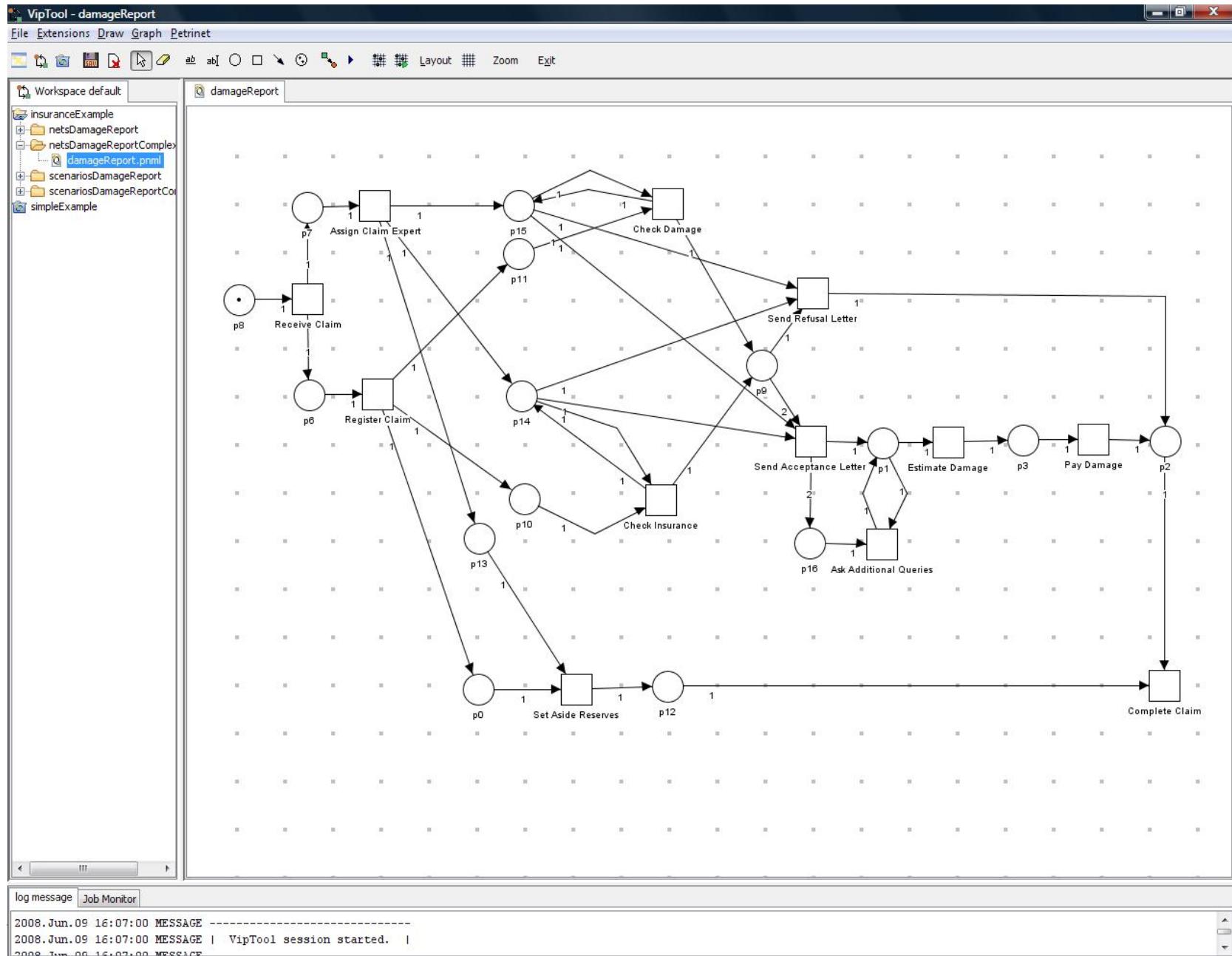


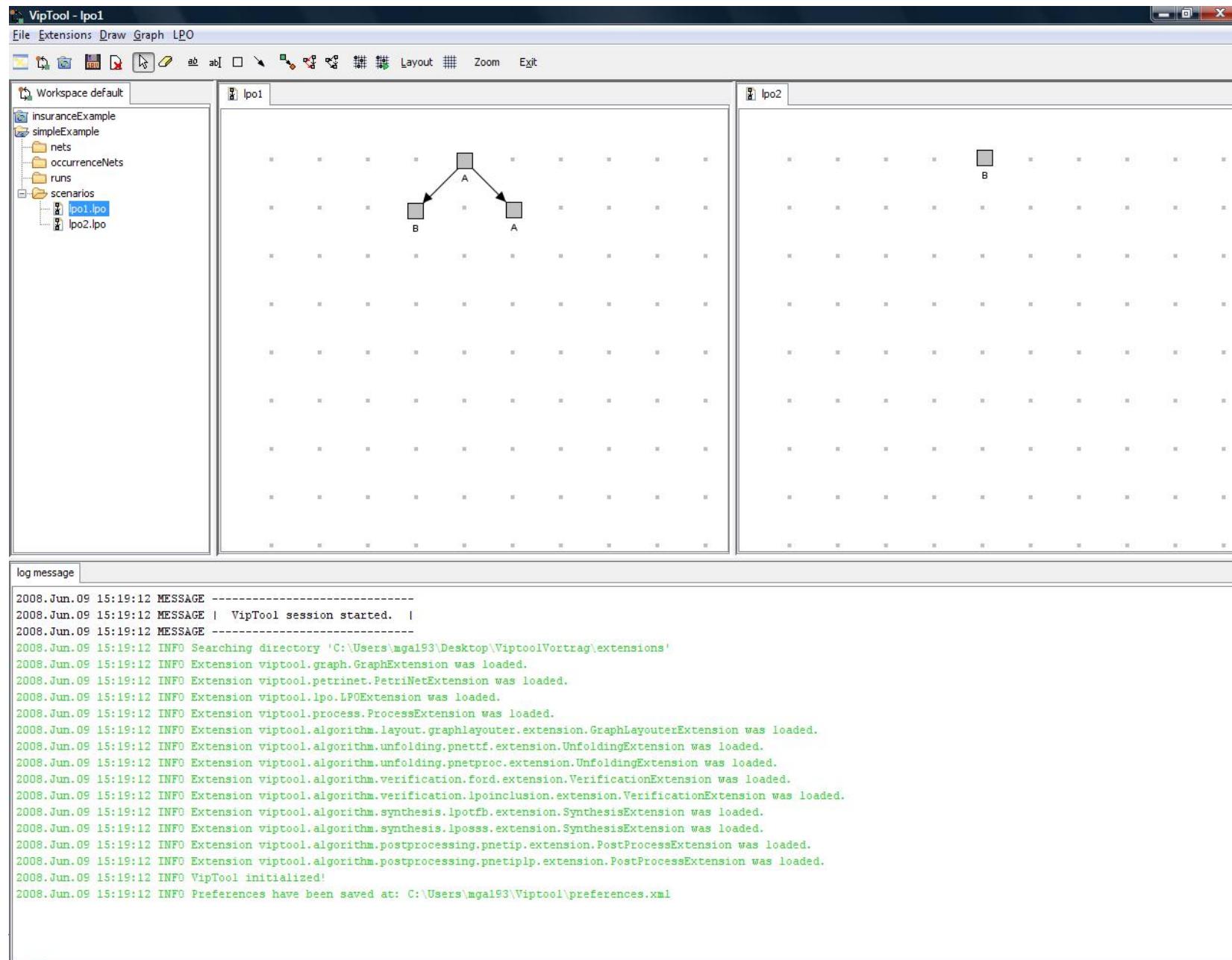
partially ordered runs

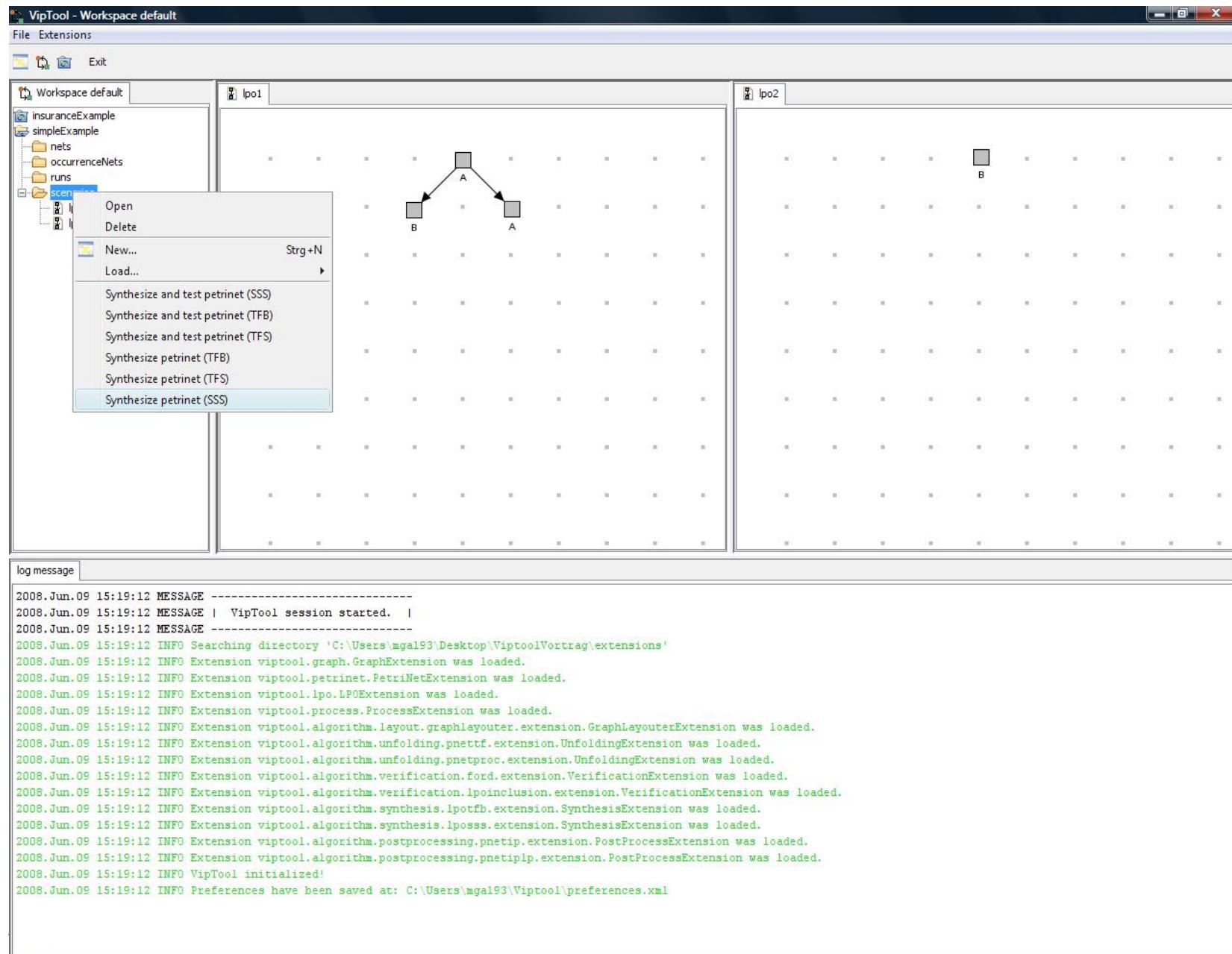


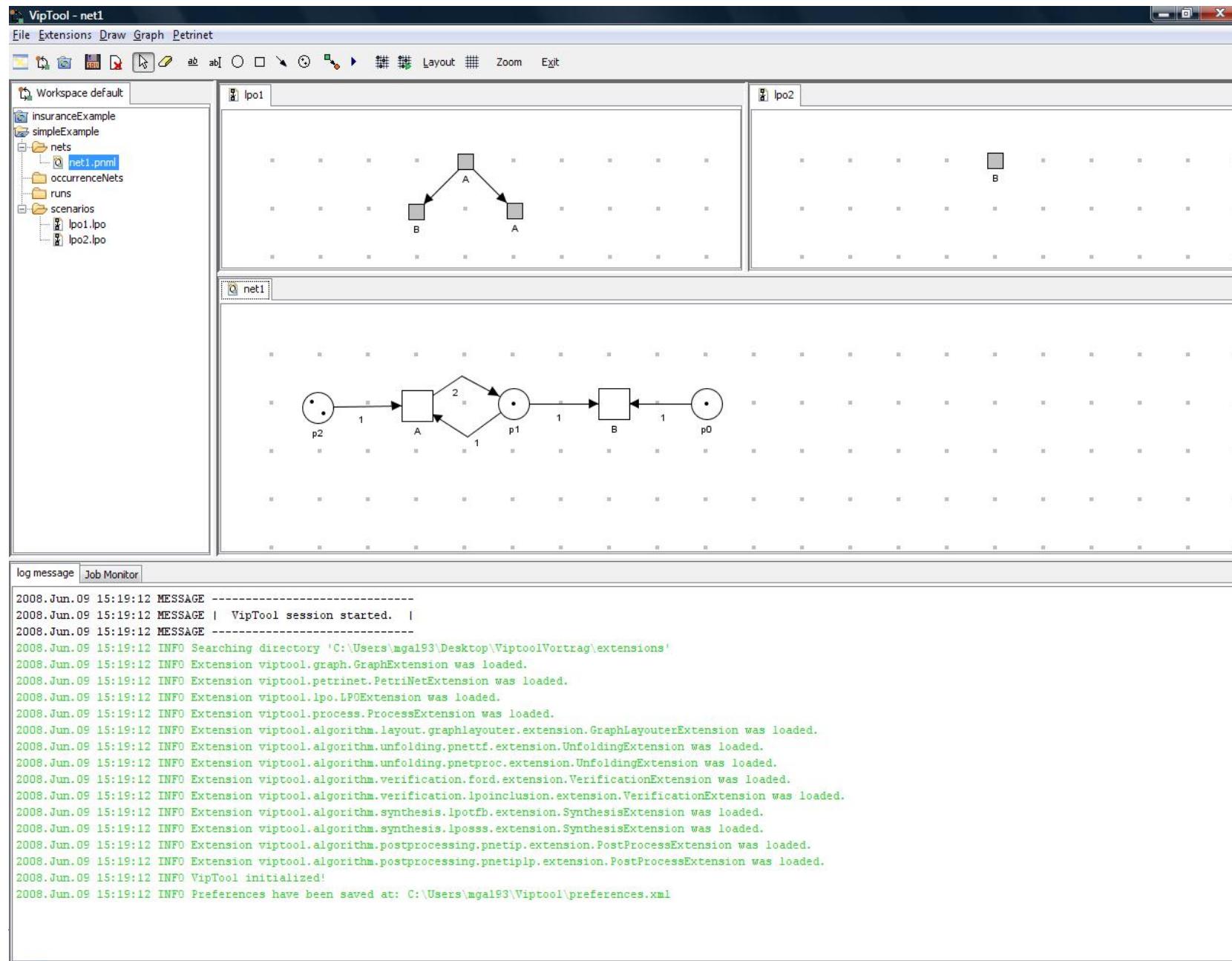


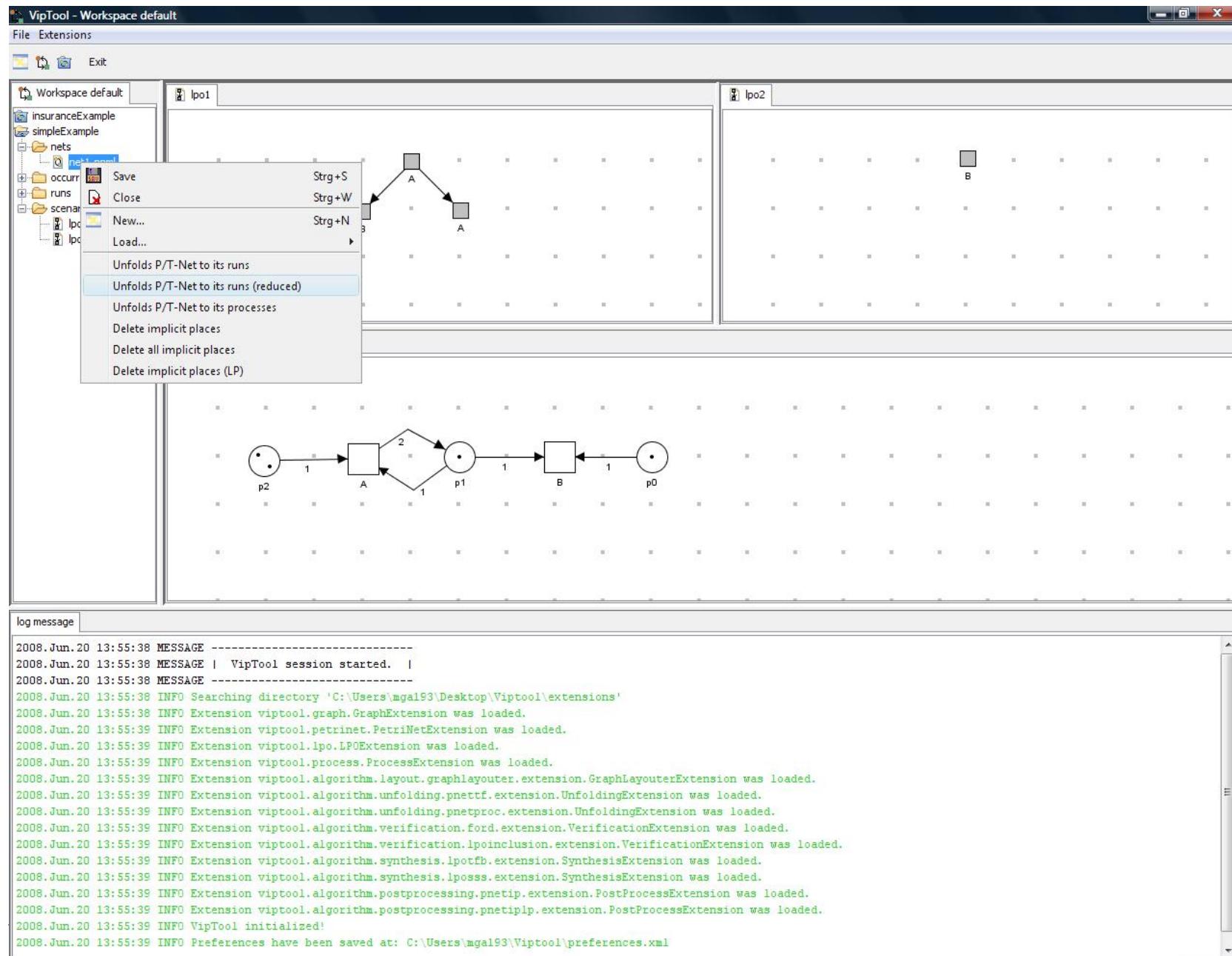


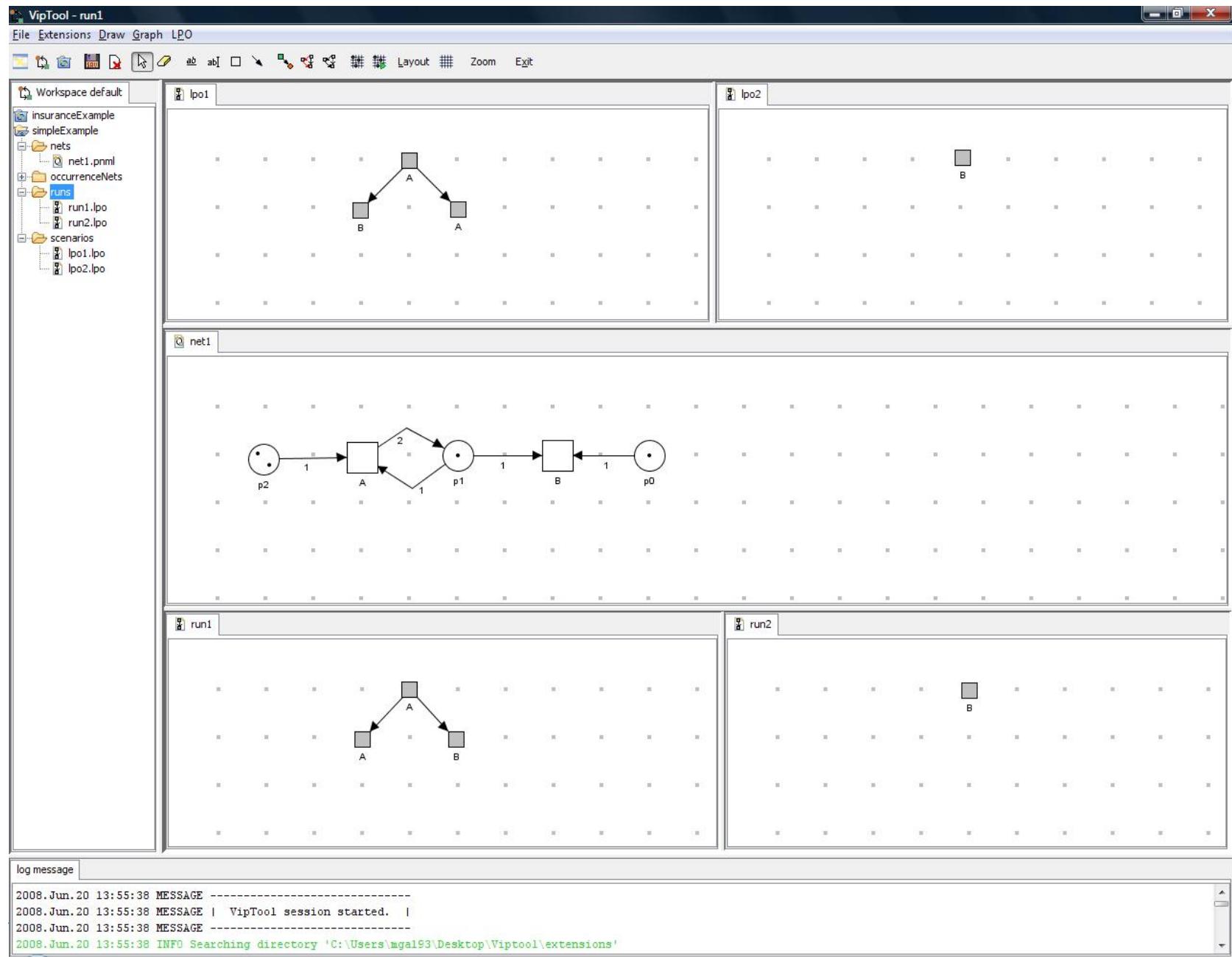


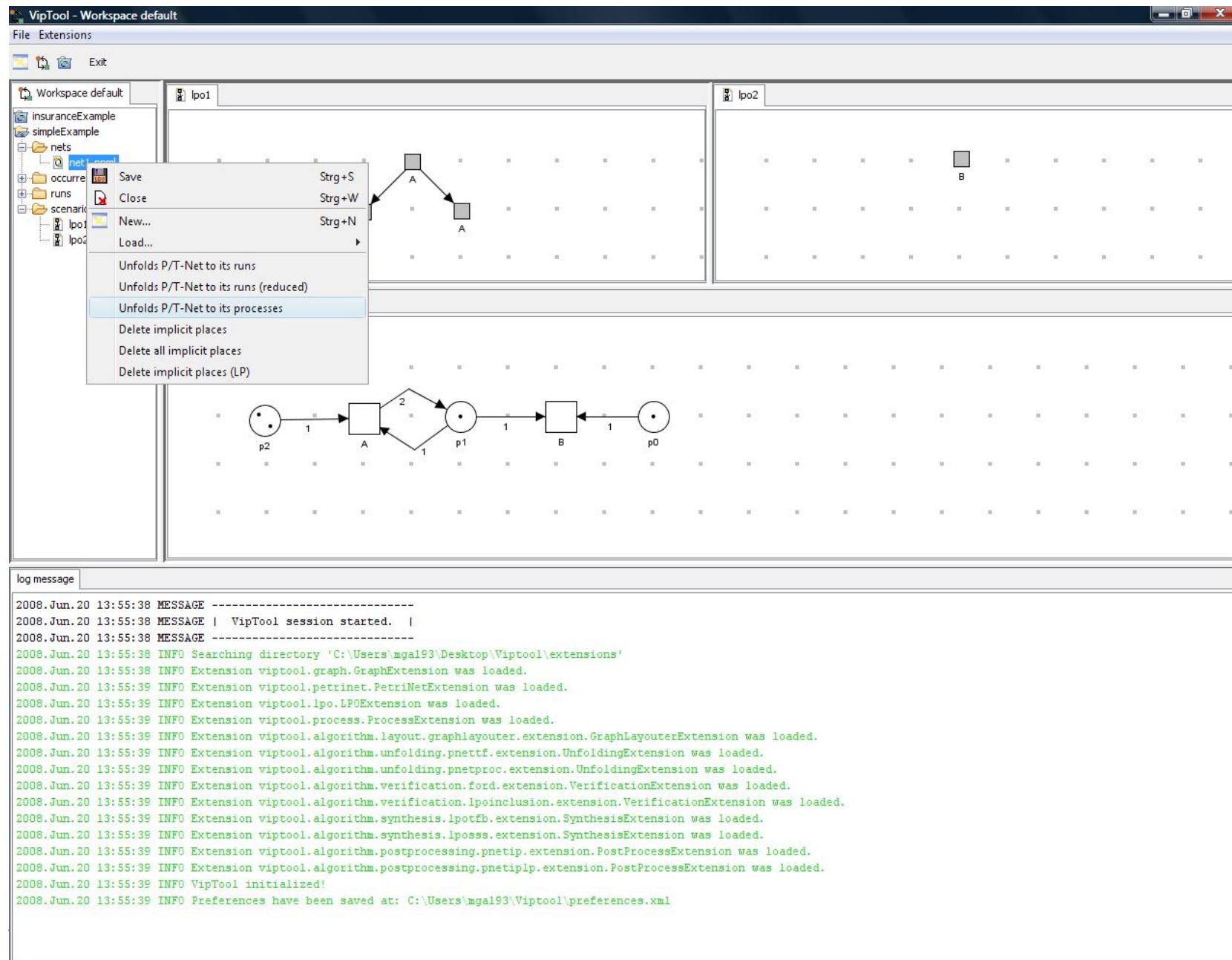


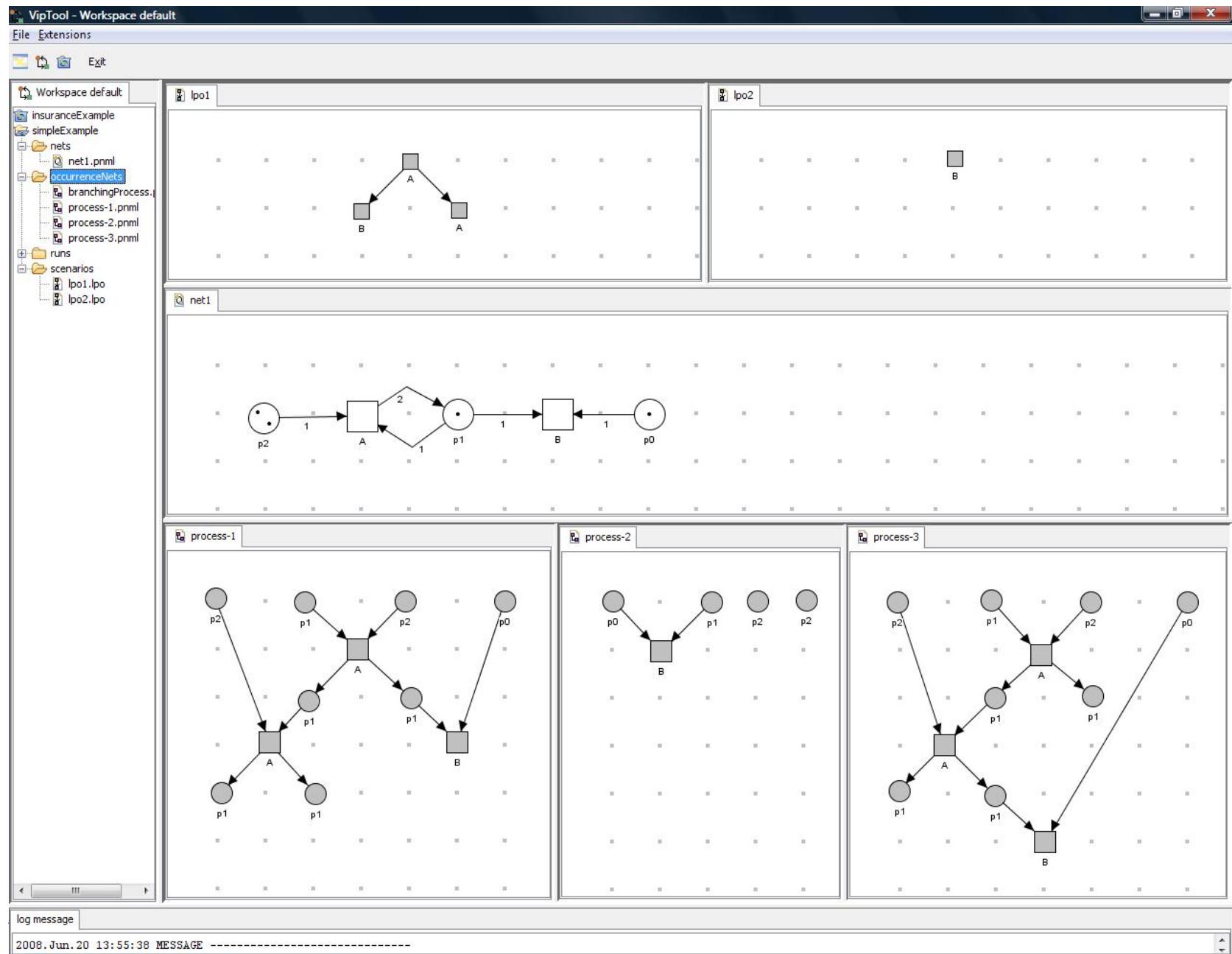




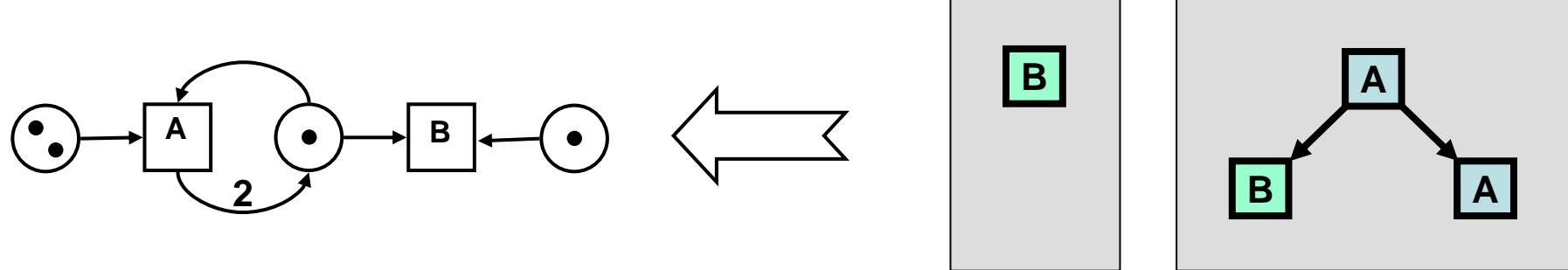




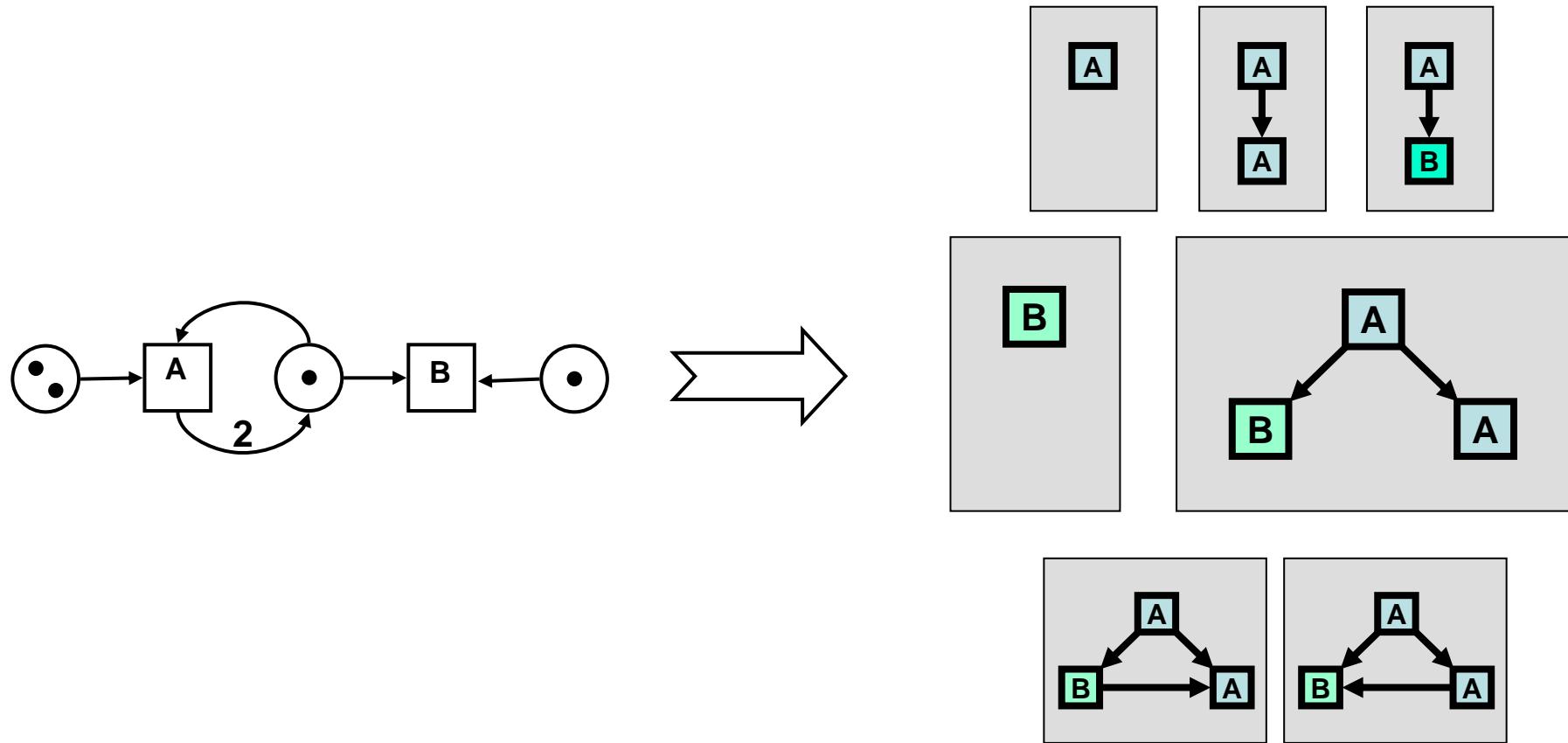


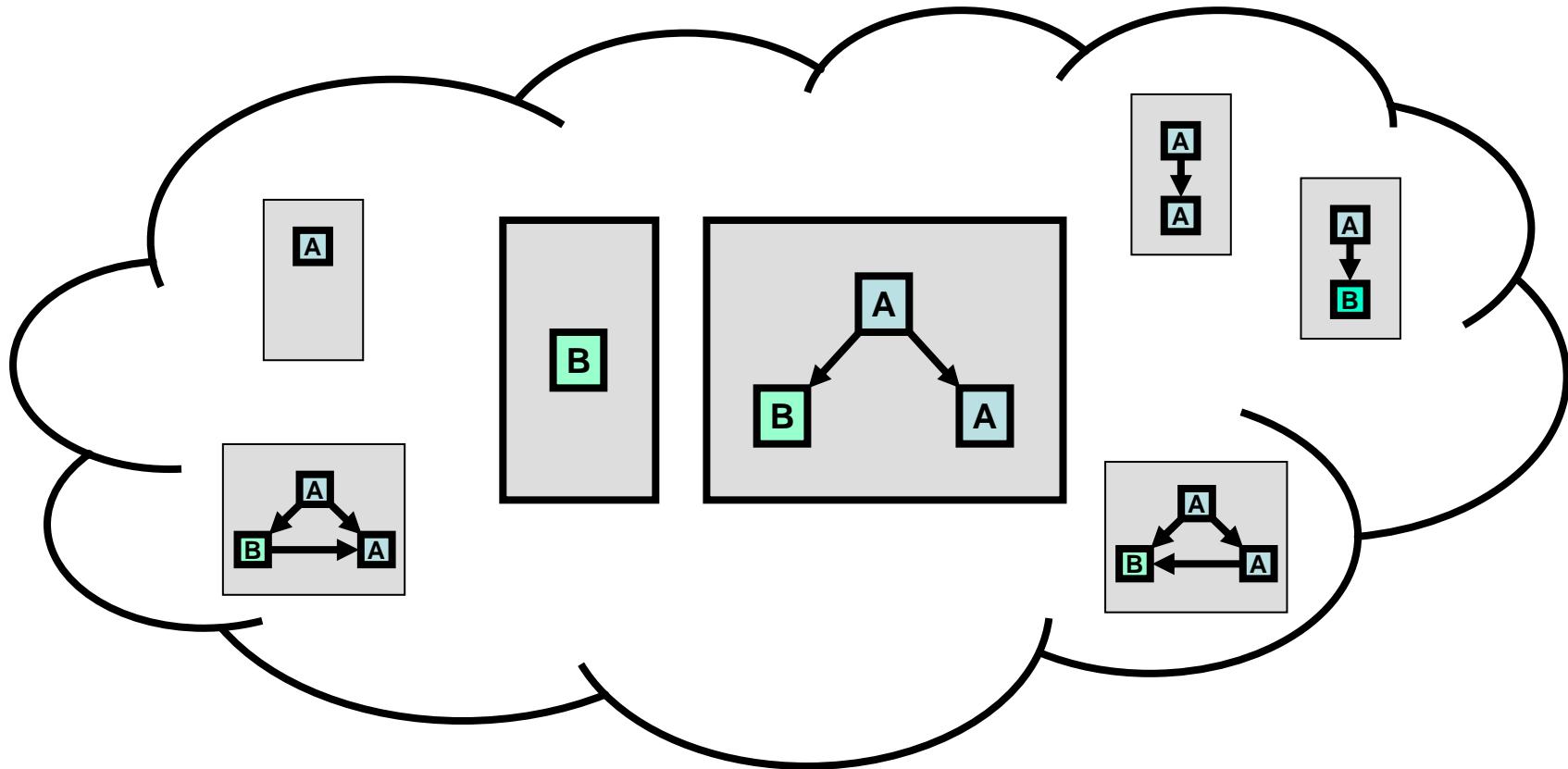


Synthesis Algorithm

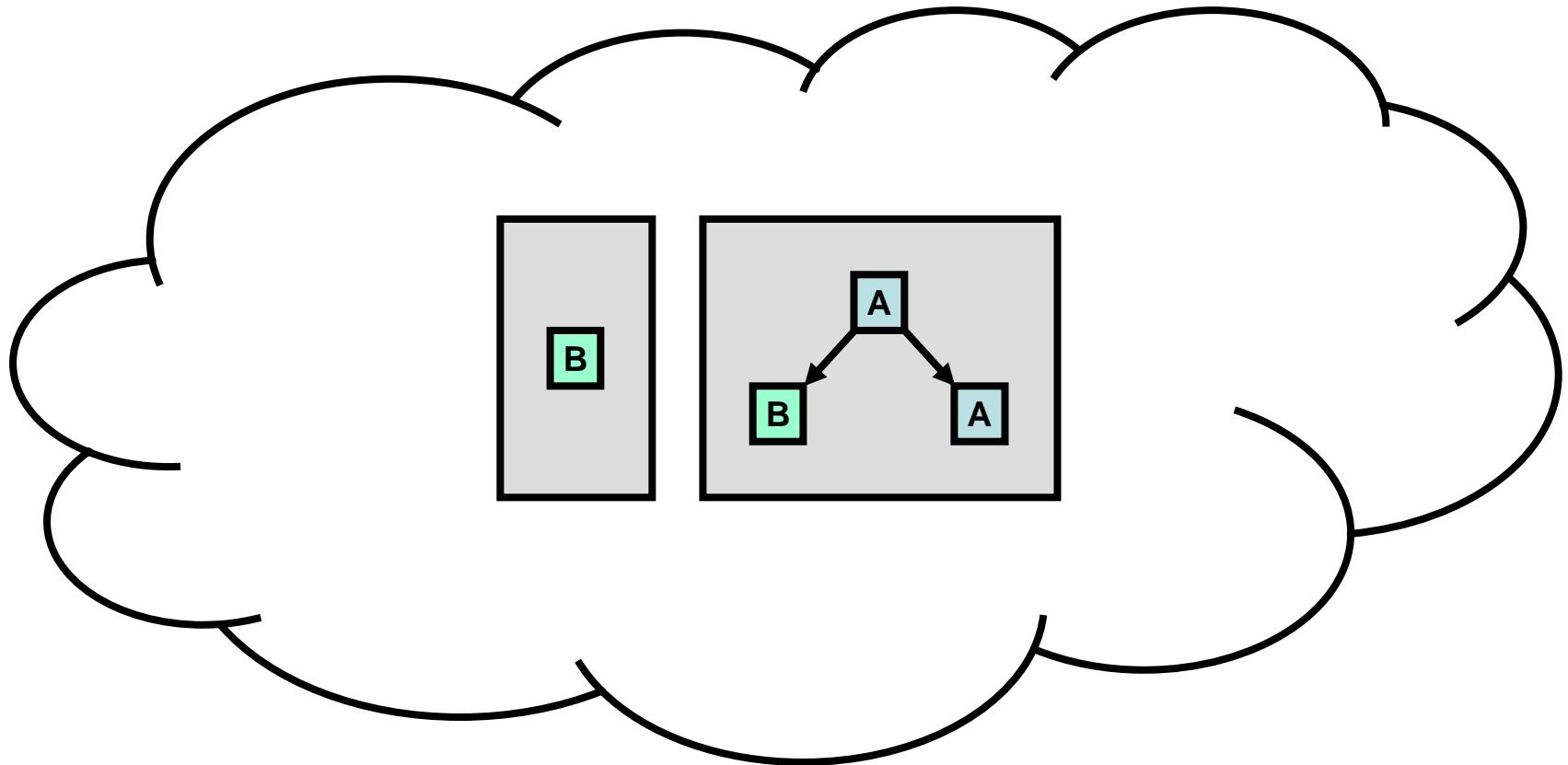


Prefixes and Sequentialization





Partial Language: finite set of LPOs, closed under
prefixing and sequalization



Partial Language: ... represented by
maximal LPOs with minimal order

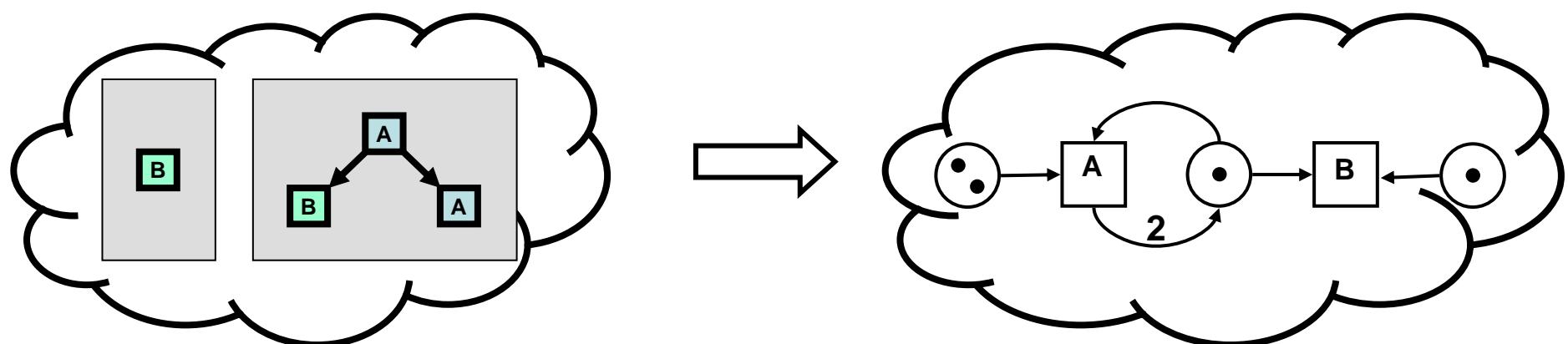
Synthese

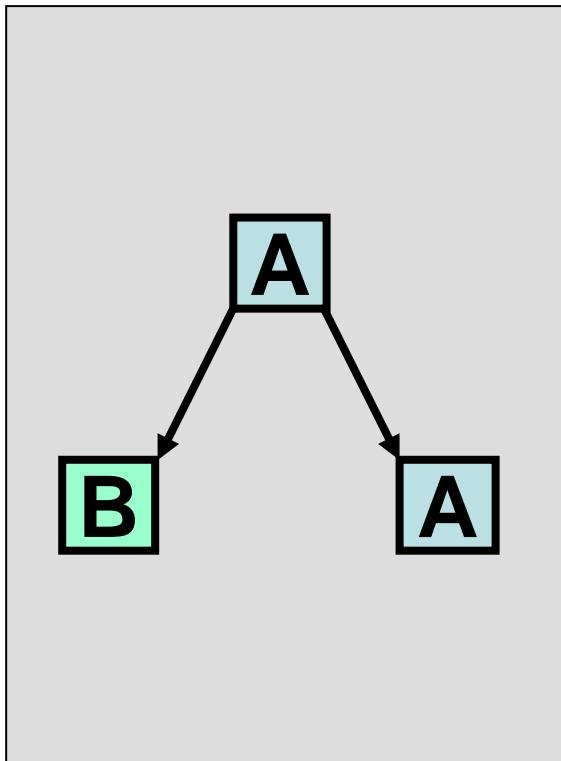
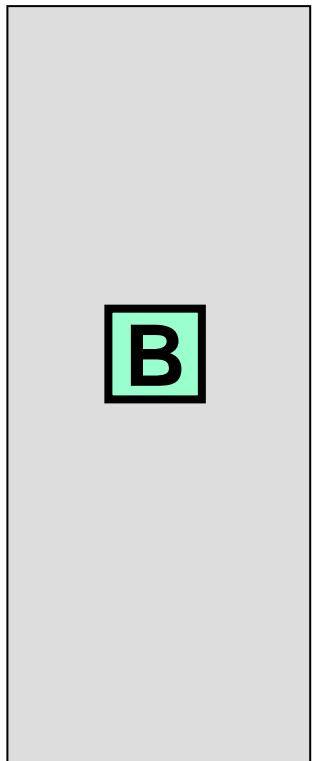
given:

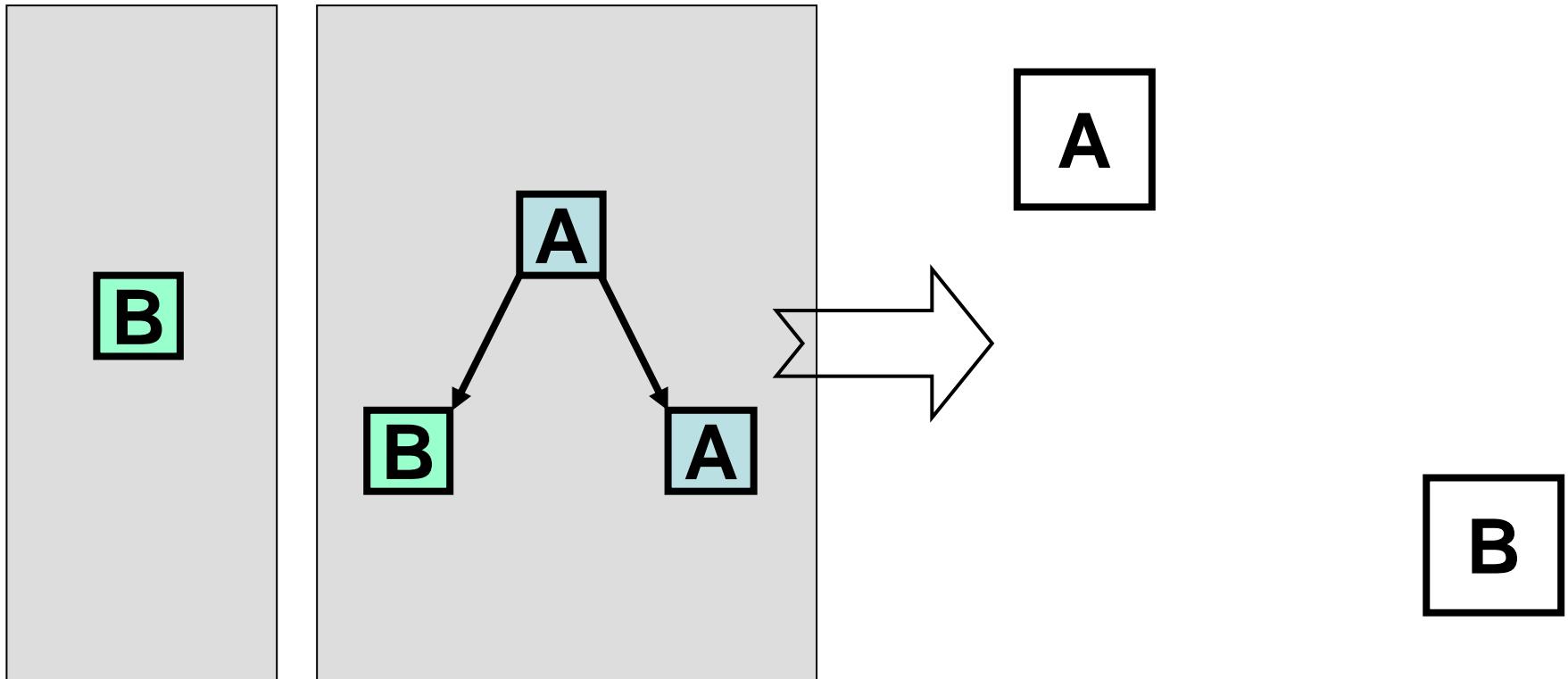
A finite partial language L

searched:

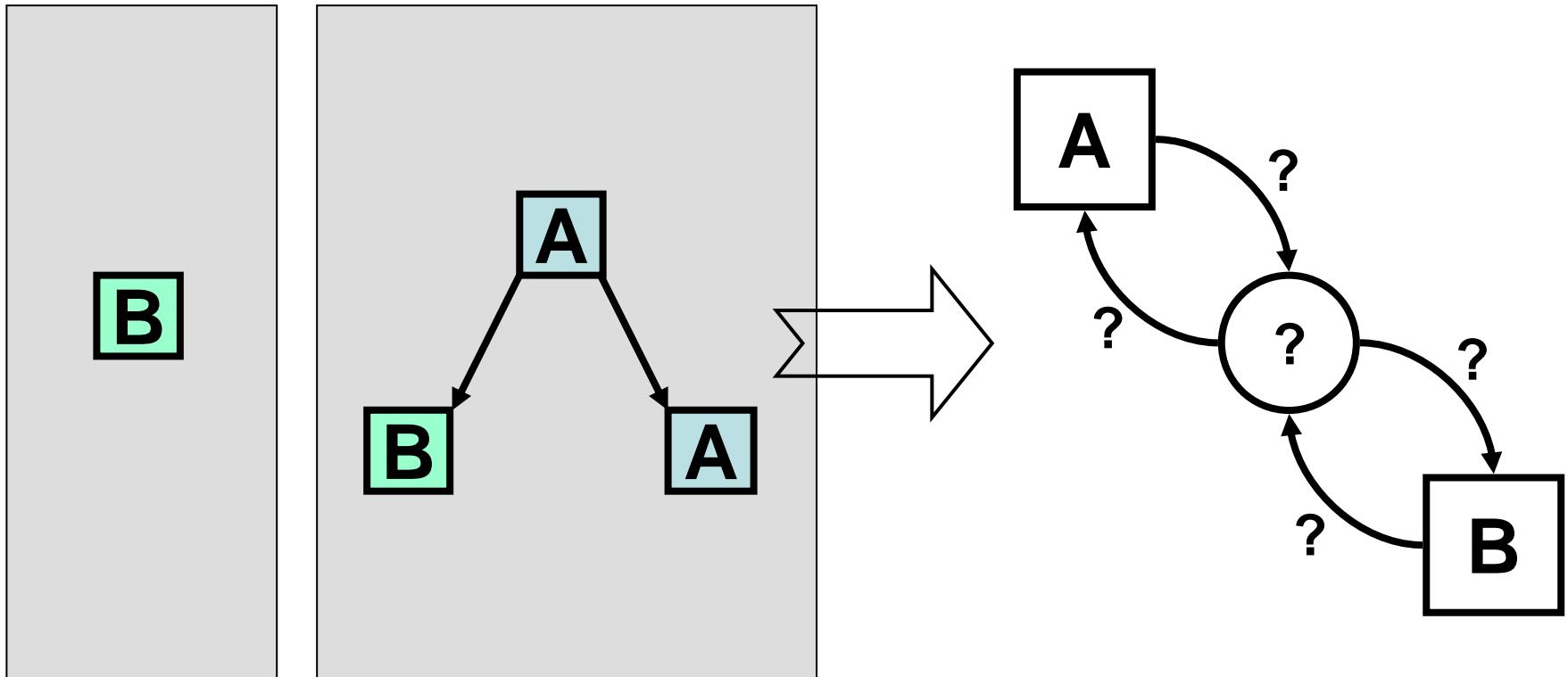
Petri net N with behaviour L



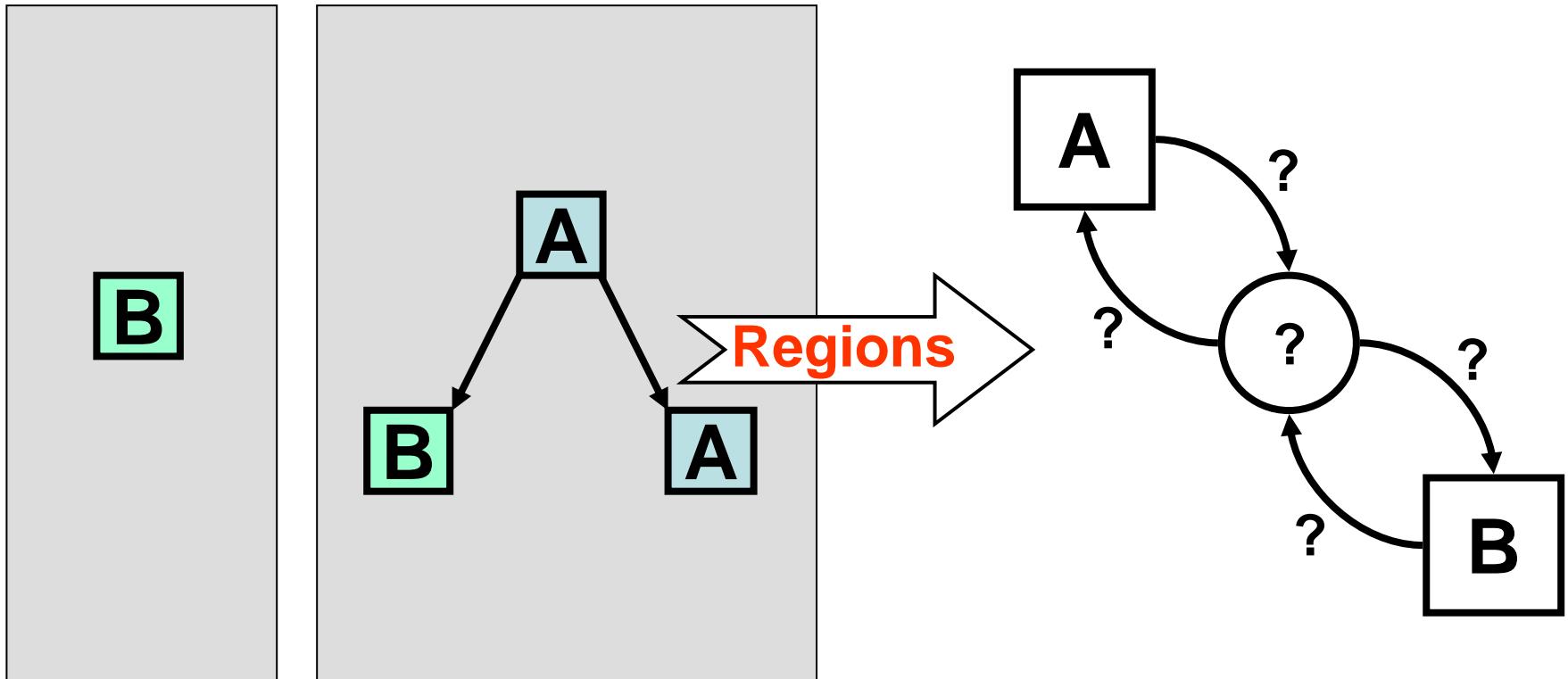




event labels \Rightarrow transitions of the net

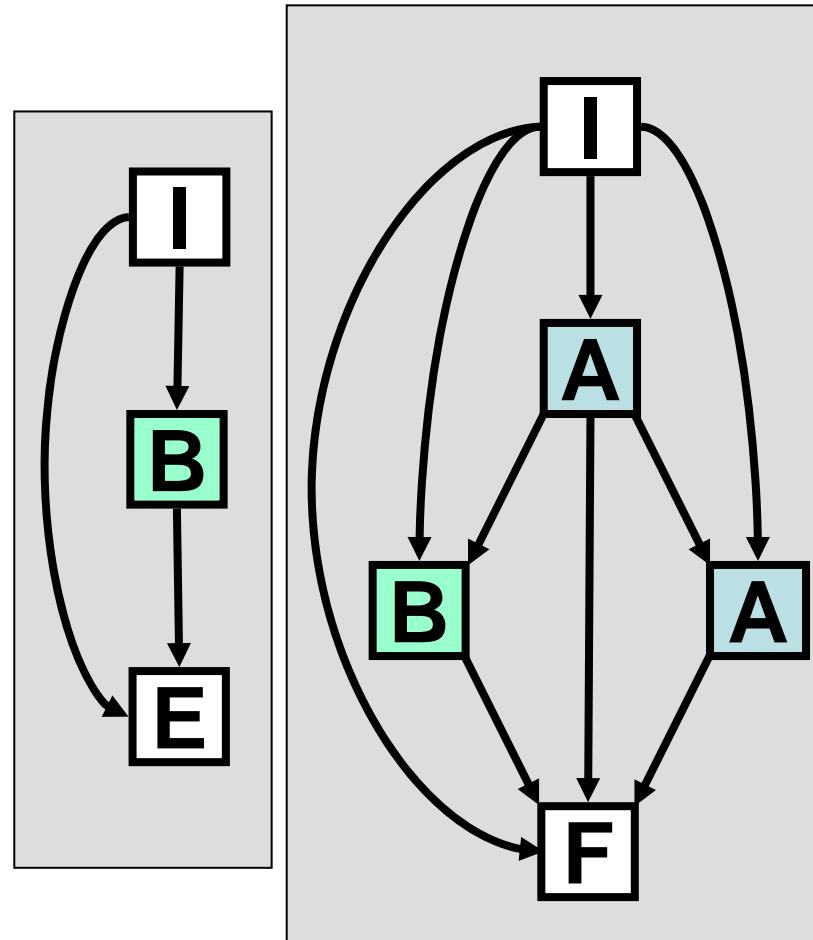


how do we find the places ?



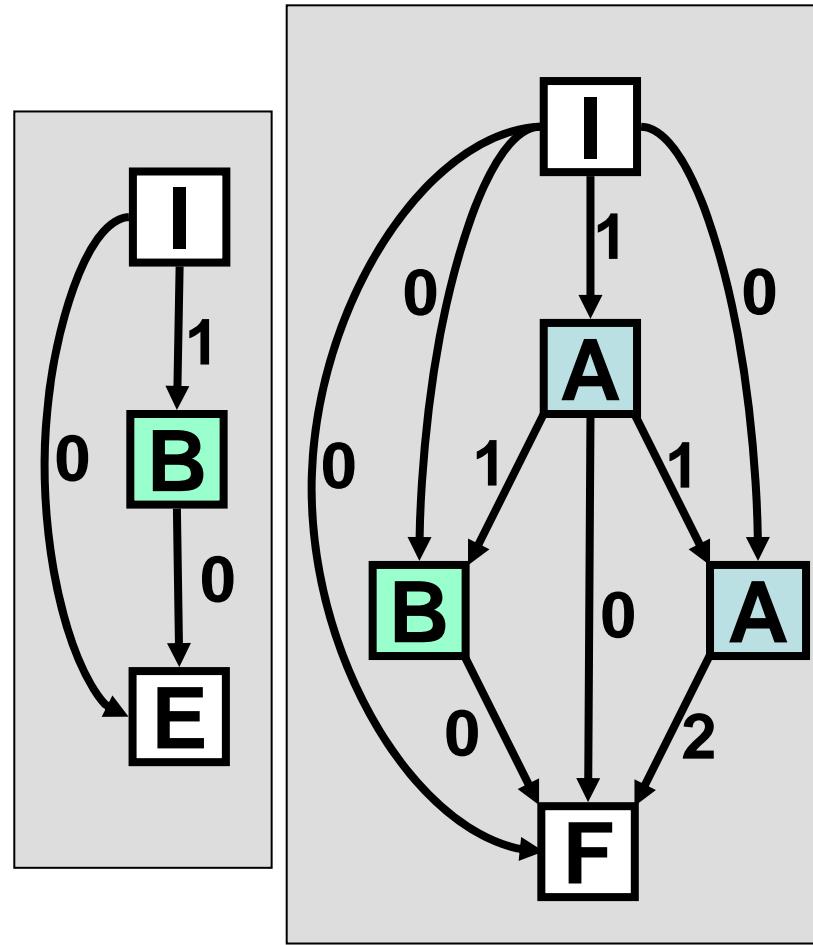
how do we find the places ?

token flow regions



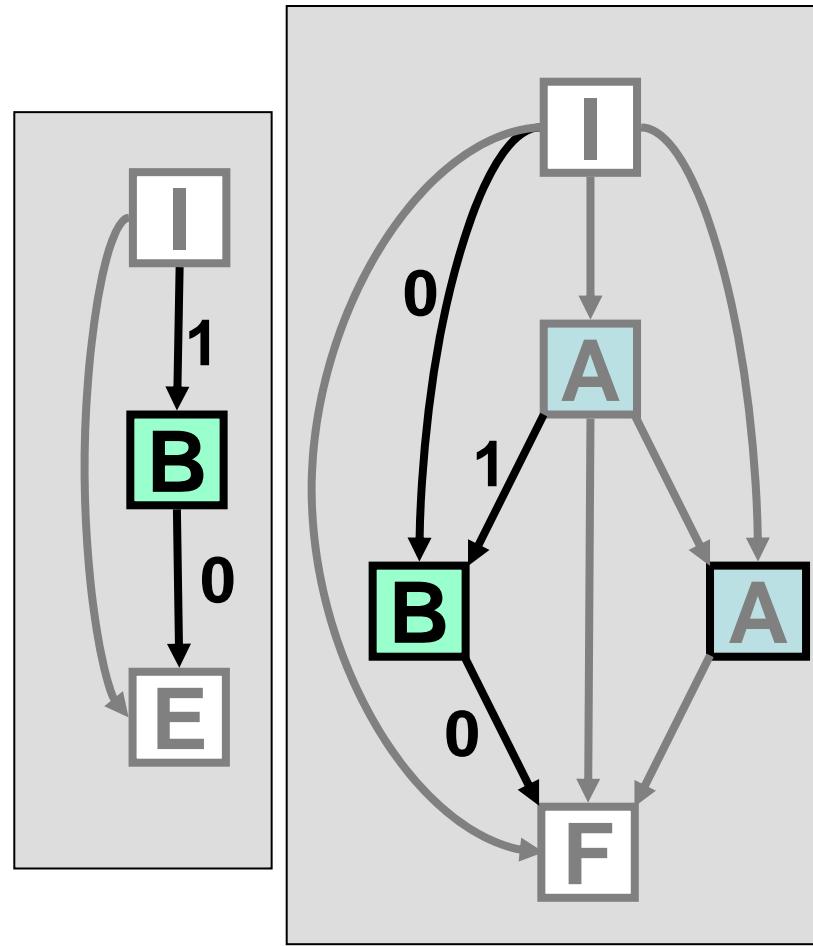
addition of **initial event (I)** (generates initial marking)
and **final events (E, F)** (consume final marking)

token flow regions



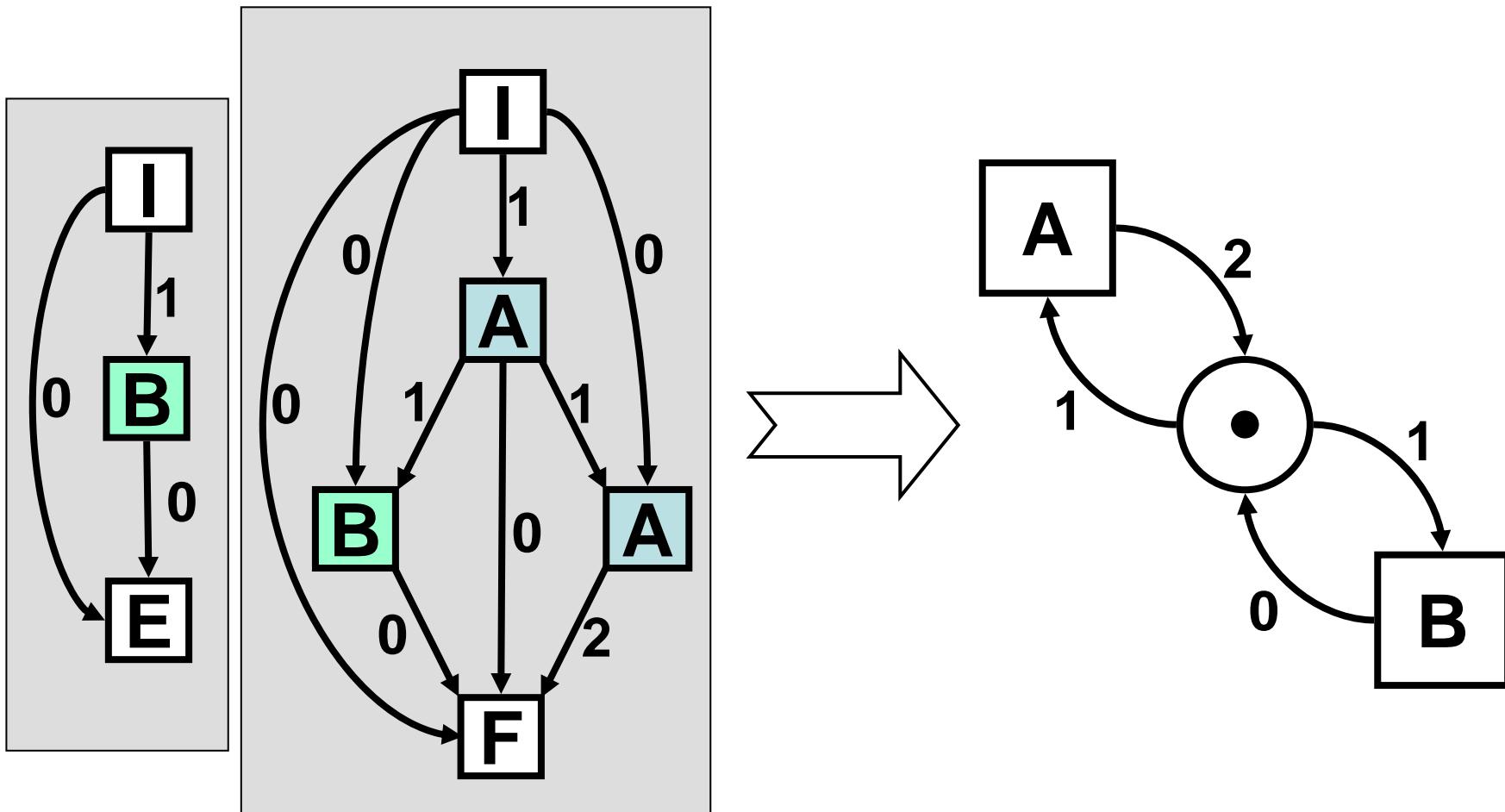
equally labelled events have equal input and output flow

token flow regions



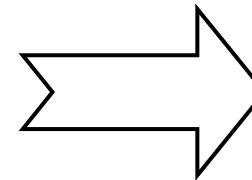
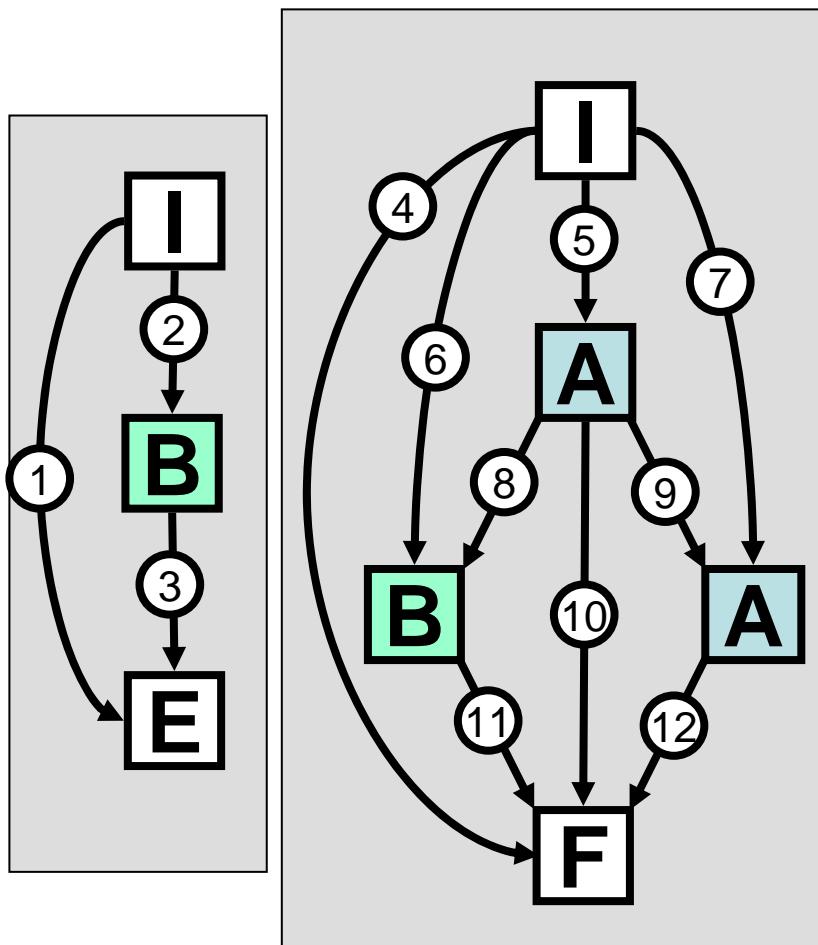
equally labelled events have equal input and output flow

token flow regions



each token flow region generates a possible place

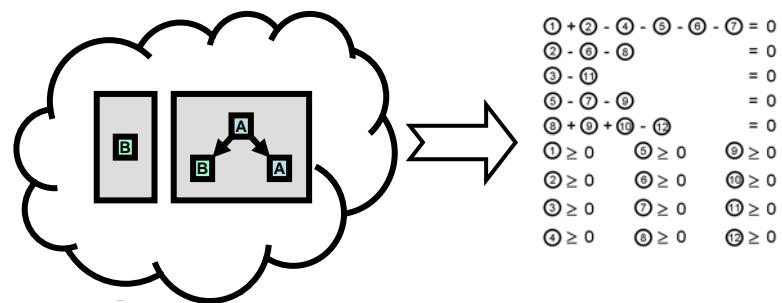
token flow regions



$$\begin{array}{lll} \textcircled{1} + \textcircled{2} - \textcircled{4} - \textcircled{5} - \textcircled{6} - \textcircled{7} = 0 & & \\ \textcircled{2} - \textcircled{6} - \textcircled{8} & = 0 & \\ \textcircled{3} - \textcircled{11} & = 0 & \\ \textcircled{5} - \textcircled{7} - \textcircled{9} & = 0 & \\ \textcircled{8} + \textcircled{9} + \textcircled{10} - \textcircled{12} & = 0 & \\ \textcircled{1} \geq 0 & \textcircled{5} \geq 0 & \textcircled{9} \geq 0 \\ \textcircled{2} \geq 0 & \textcircled{6} \geq 0 & \textcircled{10} \geq 0 \\ \textcircled{3} \geq 0 & \textcircled{7} \geq 0 & \textcircled{11} \geq 0 \\ \textcircled{4} \geq 0 & \textcircled{8} \geq 0 & \textcircled{12} \geq 0 \end{array}$$

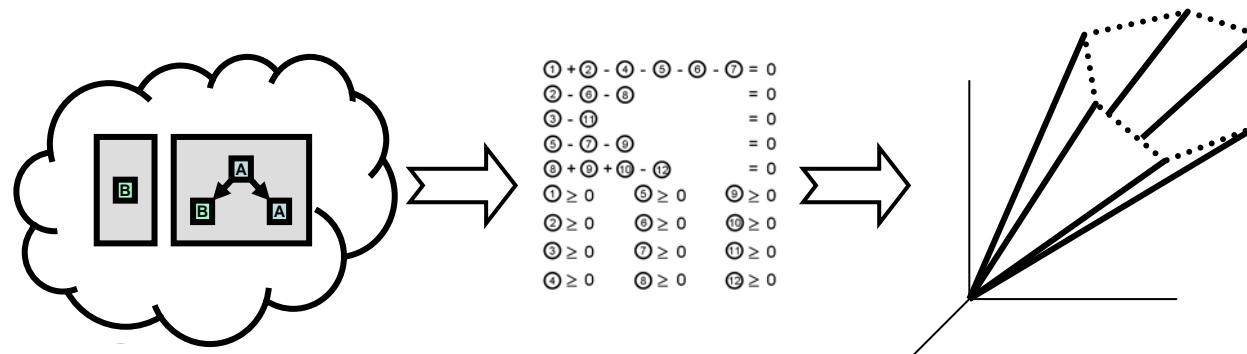
each token flow region is
an integral solution of an inequation system

which possible places should we take?



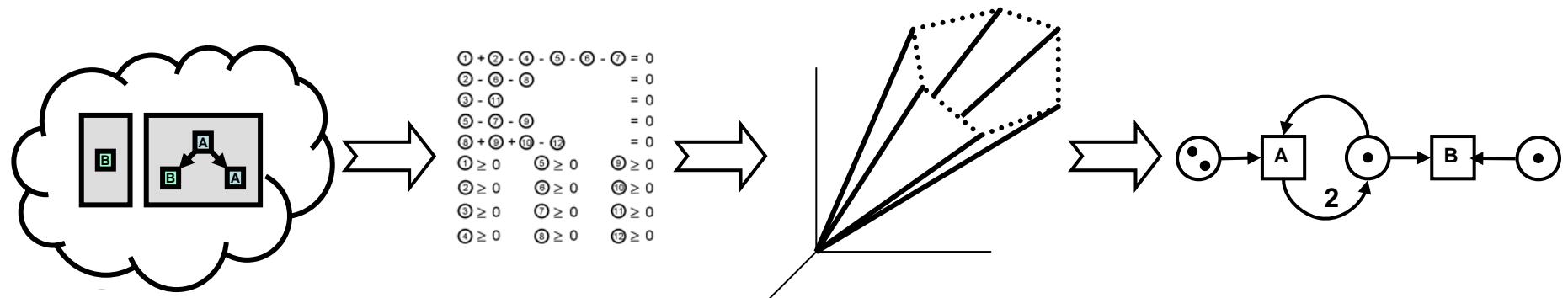
$$\begin{aligned} \textcircled{1} + \textcircled{2} - \textcircled{4} - \textcircled{5} - \textcircled{6} - \textcircled{7} &= 0 \\ \textcircled{2} - \textcircled{3} - \textcircled{6} &= 0 \\ \textcircled{3} - \textcircled{1} &= 0 \\ \textcircled{5} - \textcircled{7} - \textcircled{9} &= 0 \\ \textcircled{6} + \textcircled{8} + \textcircled{10} - \textcircled{12} &= 0 \\ \textcircled{1} \geq 0 &\quad \textcircled{3} \geq 0 \quad \textcircled{9} \geq 0 \\ \textcircled{2} \geq 0 &\quad \textcircled{8} \geq 0 \quad \textcircled{10} \geq 0 \\ \textcircled{3} \geq 0 &\quad \textcircled{7} \geq 0 \quad \textcircled{11} \geq 0 \\ \textcircled{4} \geq 0 &\quad \textcircled{6} \geq 0 \quad \textcircled{12} \geq 0 \end{aligned}$$

which possible places should we take?



solution space of the inequation system:
- pointed polyhedral cone
- generated by a finite set of rays

which possible places should we take?



this set of rays generates a finite set of places

Complete Algorithm

for each label generate a transition

generate inequation system for possible places

calculate all rays of the solution space

for each place, generate a place

If the synthesis problem has a solution,
then the synthesized net is a solution

If the synthesis problem has no solution,
then the synthesized net is a best upper approximation



Partial Orders Fit For Work

Jörg Desel

FernUniversität in Hagen